1 Appendix A: Delayed Event Time Scenarios

1.1 Simulation Notation

The following are considered without left truncation:

- T_e is the true event time
- $T_c = T_{rl}$ is the true censoring time and the record length
- $T_{obs} = min(T_e, T_c) = T_d$ is the true observed time and time difference
- $E = I(T_e < T_c)$ is the event indicator
- ϵ is the delayed event time

1.1.1 Simulation 1 - Delayed Diagnosis

- $\widetilde{T}_d = \widetilde{T}_{obs} = min(T_e + \epsilon, T_c)$ is the observed time with delayed event time
- $\widetilde{E} = I(T_e + \epsilon < T_c)$ is the observed event indicator with delayed event time

There are three cases in which subjects can be partitioned once delayed event time is added to their true event time:

Case 1. No error: $\widetilde{E} = 0$ and $T_c < T_e < T_e + \epsilon \Rightarrow \widetilde{T}_d = T_d$ and $\widetilde{E} = E = 0$

Case 2. Delayed event time leads to a misclassfied event status:
$\widetilde{E} = 0$ and $T_e < T_c < T_e + \epsilon \Rightarrow T_d < \widetilde{T}_d < T_d + \epsilon$ and $\widetilde{E} \neq E$

Case 3. Correct event indicator but incorrect time-to-event:
$\widetilde{E} = 1$ and $T_e + \epsilon < T_c \Rightarrow \widetilde{T}_d = T_d + \epsilon$ and $\widetilde{E} = E = 1$

1.1.2 Simulation 2 - Baseline Shifted

- $\bar{T}_d = min(T_e - \epsilon, T_c - \epsilon)$ is the observed time with delayed event time
- $\bar{E} = I(T_e - \epsilon < T_c - \epsilon)$ is the event indicator with delayed event time

Hence $\bar{T}_d = T_d - \epsilon$ and $\bar{E} \equiv E$

1.2 Simulation 1 - Delayed Diagnosis

1.2.1 LRM_{obs}

Assuming we model $f(\widetilde{T}_d)$ linearly:

$$
\begin{aligned}
&logit\left[P(\widetilde{E}=1|Z,\mathbf{X},\widetilde{T}_d)\right] \\
&= \beta_0+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3\widetilde{T}_d \\
&= \begin{cases} \beta_0+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_d & \text{if case 1} \\ \beta_0+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_c & \text{if case 2} \\ \beta_0+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_d+\beta_3\epsilon & \text{if case 3} \end{cases} \\
&\Rightarrow \begin{cases} = logit\left[P(E=1|Z,\mathbf{X},\widetilde{T}_d)\right] & \text{if case 1} \\ \neq logit\left[P(E=1|Z,\mathbf{X},\widetilde{T}_d)\right] & \text{if case 2} \\ = logit\left[P(E=1|Z,\mathbf{X},\widetilde{T}_d)\right] & \text{if case 3} \end{cases} \\
&\Rightarrow \begin{cases} \text{if case 1} \Rightarrow \begin{cases} = \beta_0+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_d & \text{if no delayed event time} \\ = \beta_0+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_d & \text{if delayed event time independent} \\ = \beta_0+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_d & \text{if delayed event time depends on Z} \\ = \beta_0+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_d & \text{if delayed event time depends on } \mathbf{X} \end{cases} \\ \text{if case 2} \Rightarrow \neq logit\left[P(E=1|Z,\mathbf{X},\widetilde{T}_d)\right] \\ \text{if case 3} \Rightarrow \begin{cases} = \beta_0+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_d & \text{if no delayed event time} \\ = (\beta_0+\beta_3\epsilon)+\beta_1 Z+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_d & \text{if delayed event time independent} \\ = \beta_0+(\beta_1 Z+\beta_3 g(Z))+\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 T_d & \text{if delayed event time depends on Z} \\ = \beta_0+\beta_1 Z+(\boldsymbol{\beta}_2'\mathbf{X}+\beta_3 g(\mathbf{X}))+\beta_3 T_d & \text{if delayed event time depends on } \mathbf{X} \end{cases} \end{cases}
\end{aligned}
$$

where $g(Z)$ is the delayed event time as a function of Z and $g(\mathbf{X})$ is the delayed event time as a function of $\mathbf{X}$. Then we have:

$$
\begin{aligned}
&logit\left[P(\widetilde{E}=1|Z,\mathbf{X},\widetilde{T}_d)\right] \\
&\Rightarrow \begin{cases}
\text{if case 1} \Rightarrow \begin{cases}
= logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if no delayed event time} \\
= logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if delayed event time independent} \\
= logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if delayed event time depends on Z} \\
= logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if delayed event time depends on } \mathbf{X}
\end{cases} \\
\text{if case 2} \Rightarrow \neq logit\left[P(E=1|Z,\mathbf{X},T_d)\right] \\
\text{if case 3} \Rightarrow \begin{cases}
= logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if no delayed event time} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if delayed event time independent} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if delayed event time depends on Z} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if delayed event time depends on } \mathbf{X}
\end{cases}
\end{cases}
\end{aligned}
$$

1.2.2 LRM_u

$$
logit\left[P(\widetilde{E}=1|Z,\mathbf{X})\right] = \beta_0 + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} \Rightarrow \begin{cases}
= logit\left[P(E=1|Z,\mathbf{X})\right] & \text{if case 1} \\
\neq logit\left[P(E=1|Z,\mathbf{X})\right] & \text{if case 2} \\
= logit\left[P(E=1|Z,\mathbf{X})\right] & \text{if case 3}
\end{cases}
$$

1.2.3 LRM_{rl}

Assuming we model $f(\widetilde{T}_{rl})$ linearly:

$$
logit\left[P(\widetilde{E}=1|Z,\mathbf{X},T_{rl})\right] = \beta_0 + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3 T_{rl} \Rightarrow \begin{cases}
= logit\left[P(E=1|Z,\mathbf{X},T_{rl})\right] & \text{if case 1} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_{rl})\right] & \text{if case 2} \\
= logit\left[P(E=1|Z,\mathbf{X},T_{rl})\right] & \text{if case 3}
\end{cases}
$$

1.3 Simulation 2 - Baseline Shifted

1.3.1 LRM_{obs}

Assuming we model $f(\bar{T}_d)$ linearly:

$$
\begin{aligned}
&logit\left[P(\bar{E}=1|Z,\mathbf{X},\bar{T}_d)\right] \\
&= \beta_0 + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3\bar{T}_d \\
&= \beta_0 + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3(T_d - \epsilon) \\
&= \beta_0 + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3 T_d - \beta_3\epsilon \\
&\Rightarrow \begin{cases}
= \beta_0 + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3 T_d & \text{if no delayed event time} \\
= (\beta_0 - \beta_3\epsilon) + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3 T_d & \text{if delayed event time independent} \\
= \beta_0 + (\beta_1 Z - \beta_3 g(Z)) + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3 T_d & \text{if delayed event time depends on Z} \\
= \beta_0 + \beta_1 Z + (\boldsymbol{\beta}_2'\mathbf{X} - \beta_3 g(\mathbf{X})) + \beta_3 T_d & \text{if delayed event time depends on } \mathbf{X}
\end{cases} \\
&\Rightarrow \begin{cases}
= logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if no delayed event time} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if delayed event time independent} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if delayed event time depends on Z} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_d)\right] & \text{if delayed event time depends on } \mathbf{X}
\end{cases}
\end{aligned}
$$

where $g(Z)$ is the delayed event time as a function of Z and $g(\mathbf{X})$ is the delayed event time as a function of $\mathbf{X}$.

1.3.2 LRM_u

$$logit\left[P(\bar{E}=1|Z,\mathbf{X})\right] = \beta_0 + \beta_1 Z + \beta_2'\mathbf{X} = logit\left[P(E=1|Z,\mathbf{X})\right]$$

1.3.3 LRM_{rl}

Assuming we model $f(\bar{T}_{rl})$ linearly, where $\bar{T}_{rl} = T_{rl} - \epsilon = T_c - \epsilon$:

$$
\begin{aligned}
&logit\left[P(\bar{E}=1|Z,\mathbf{X},T_{rl})\right] \\
&= \beta_0 + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3(T_{rl}-\epsilon) \\
&\Rightarrow \begin{cases}
= \beta_0 + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3 T_{rl} & \text{if no delayed event time} \\
= (\beta_0 - \beta_3\epsilon) + \beta_1 Z + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3 T_{rl} & \text{if delayed event time independent} \\
= \beta_0 + (\beta_1 Z - \beta_3 g[Z]) + \boldsymbol{\beta}_2'\mathbf{X} + \beta_3 T_{rl} & \text{if delayed event time depends on Z} \\
= \beta_0 + \beta_1 Z + (\boldsymbol{\beta}_2'\mathbf{X} - \beta_3 g[\mathbf{X}]) + \beta_3 T_{rl} & \text{if delayed event time depends on } \mathbf{X}
\end{cases} \\
&\Rightarrow \begin{cases}
= logit\left[P(E=1|Z,\mathbf{X},T_{rl})\right] & \text{if no delayed event time} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_{rl})\right] & \text{if delayed event time independent} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_{rl})\right] & \text{if delayed event time depends on Z} \\
\neq logit\left[P(E=1|Z,\mathbf{X},T_{rl})\right] & \text{if delayed event time depends on } \mathbf{X}
\end{cases}
\end{aligned}
$$

2 Appendix B: Additional Tables

Phenotype	Phecode	No Delayed Event Time Event Rate	Significant SNPs Event Rate	Non-significant SNPs Event Rate	Independent Event Rate	Sex Event Rate
Cancer of bronchus; lung	165.1	2.74%	1.95%	2.06%	1.78%	1.74%
Cancer of prostate *	185	6.52%	6.11%	5.80%	5.78%	-
Hypothyroidism	244	11.74%	10.94%	10.93%	10.64%	10.73%
Type 2 diabetes	250.2	14.33%	13.23%	13.23%	12.79%	12.73%
Vitamin D deficiency	261.4	6.72%	6.16%	6.11%	6.02%	6.12%
Hypercholesterolemia	272.11	9.99%	9.64%	9.75%	9.64%	9.62%
Insomnia	327.4	4.46%	4.00%	4.14%	4.11%	4.11%
Myocardial infarction	411.2	5.61%	4.54%	4.72%	4.66%	4.59%
Coronary atherosclerosis	411.4	16.83%	15.46%	15.51%	14.99%	14.89%
Atrial fibrillation	427.21	9.93%	8.65%	8.66%	8.3%	8.19%

Table S1: Phecodes used in the GWAS application. Includes information about the phenotype, phecode, and the event rate for each delayed event time scenario.

* Analysis performed for males only.

	True Value for β_1					
	$log(1)$	$log(1.1)$	$log(1.15)$	$log(1.25)$	$log(1.5)$	$log(2)$
Simulation 1 with left truncation						
Fig. S1 (Simulation 1, left truncation, censoring depending on (x_1, x_2), removal-practice, large misclassification)						
No delayed error	-0.002	0.000	0.001	0.002	0.005	0.009
Independent error	-0.001	-0.013	-0.020	-0.033	-0.068	-0.136
Exposure-dependent error	1.814	1.834	1.842	1.861	1.904	1.984
Confounder-dependent error	-0.003	0.003	0.005	0.008	0.011	0.005
Covariate-dependent error	-0.005	-0.018	-0.025	-0.037	-0.069	-0.131
Fig. S2 (Simulation 1, left truncation, censoring depending on (x_1, x_2), censor-practice, large misclassification)						
No delayed error	-0.003	-0.048	-0.071	-0.118	-0.234	-0.467
Independent error	0.000	-0.015	-0.022	-0.038	-0.078	-0.157
Exposure-dependent error	1.671	1.667	1.662	1.653	1.622	1.530
Confounder-dependent error	-0.006	-0.020	-0.029	-0.048	-0.098	-0.214
Covariate-dependent error	-0.006	-0.037	-0.052	-0.082	-0.158	-0.309
Fig. S3 (Simulation 1, left truncation, censoring depending on (x_1, x_2, z), removal-practice, large misclassification)						
No delayed error	-0.002	-0.000	-0.000	0.001	0.005	0.009
Independent error	-0.004	-0.017	-0.023	-0.037	-0.072	-0.140
Exposure-dependent error	1.870	1.889	1.897	1.914	1.955	2.031
Confounder-dependent error	0.003	0.008	0.010	0.012	0.015	0.010
Covariate-dependent error	-0.005	-0.016	-0.022	-0.036	-0.066	-0.125
Fig. S4 (Simulation 1, left truncation, censoring depending on (x_1, x_2, z), censor-practice, large misclassification)						
No delayed error	-0.000	-0.044	-0.067	-0.112	-0.224	-0.450
Independent error	-0.004	-0.018	-0.025	-0.041	-0.081	-0.159
Exposure-dependent error	1.734	1.731	1.727	1.719	1.690	1.605
Confounder-dependent error	0.002	-0.012	-0.020	-0.038	-0.086	-0.197
Covariate-dependent error	-0.007	-0.034	-0.048	-0.079	-0.152	-0.296

	True Value for β_1					
	$log(1)$	$log(1.1)$	$log(1.15)$	$log(1.25)$	$log(1.5)$	$log(2)$
Simulation 1 without truncation						
Fig. S5 (Simulation 1, no truncation, random censoring, large misclassification)						
No delayed error	-0.004	-0.004	-0.004	-0.003	-0.002	0.001
Independent error	-0.008	-0.026	-0.034	-0.051	-0.093	-0.176
Exposure-dependent error	2.286	2.288	2.290	2.292	2.298	2.313
Confounder-dependent error	-0.002	0.001	0.002	0.003	0.005	0.009
Covariate-dependent error	-0.001	-0.008	-0.012	-0.020	-0.037	-0.074
Fig. S6 (Simulation 1, no truncation, censoring depending on (x_1, x_2), large misclassification)						
No delayed error	-0.004	-0.002	-0.002	-0.002	0.000	0.003
Independent error	-0.005	-0.017	-0.023	-0.037	-0.069	-0.137
Exposure-dependent error	1.892	1.911	1.921	1.940	1.985	2.069
Confounder-dependent error	-0.001	0.005	0.008	0.011	0.018	0.016
Covariate-dependent error	0.003	-0.007	-0.013	-0.021	-0.045	-0.089
Fig. S7 (Simulation 1, no truncation, censoring depending on (x_1, x_2, z), large misclassification)						
No delayed error	-0.004	-0.003	-0.003	-0.002	-0.000	0.002
Independent error	-0.006	-0.019	-0.025	-0.039	-0.072	-0.140
Exposure-dependent error	1.948	1.968	1.978	1.997	2.038	2.118
Confounder-dependent error	0.004	0.010	0.012	0.015	0.021	0.021
Covariate-dependent error	0.002	-0.006	-0.011	-0.020	-0.042	-0.083

	True Value for β_1					
	$log(1)$	$log(1.1)$	$log(1.15)$	$log(1.25)$	$log(1.5)$	$log(2)$
Simulation 2						
Fig. S8 (Simulation 2, random censoring, low percentage data removal)						
No delayed error	-0.003	-0.003	-0.003	-0.003	-0.002	-0.000
Independent error	-0.008	-0.007	-0.006	-0.004	-0.002	0.002
Exposure-dependent error	-0.010	-0.008	-0.007	-0.006	-0.004	-0.002
Confounder-dependent error	-0.009	-0.007	-0.006	-0.005	-0.004	-0.002
Covariate-dependent error	-0.006	-0.005	-0.004	-0.003	-0.003	0.001
Fig. S9 (Simulation 2, random censoring, large percentage data removal)						
No delayed error	-0.004	-0.004	-0.004	-0.003	-0.002	0.001
Independent error	-0.007	-0.002	-0.001	0.000	0.002	0.007
Exposure-dependent error	0.013	0.014	0.015	0.016	0.018	0.023
Confounder-dependent error	0.001	0.002	0.003	0.003	0.007	0.011
Covariate-dependent error	0.003	0.004	0.004	0.005	0.007	0.011
Fig. S10 (Simulation 2, censoring depending on (x_1,x_2), low percentage data removal)						
No delayed error	-0.004	-0.003	-0.002	-0.001	-0.000	0.003
Independent error	-0.005	-0.004	-0.003	-0.002	-0.000	0.003
Exposure-dependent error	-0.002	-0.001	-0.000	0.001	0.004	0.007
Confounder-dependent error	-0.002	-0.001	-0.000	0.000	0.003	0.007
Covariate-dependent error	0.001	0.002	0.003	0.004	0.006	0.010
Fig. S11 (Simulation 2, censoring depending on (x_1, x_2), large percentage data removal)						
No delayed error	-0.004	-0.002	-0.002	-0.002	0.000	0.003
Independent error	-0.003	-0.000	0.002	0.003	0.007	0.018
Exposure-dependent error	-0.001	0.001	0.001	0.004	0.010	0.015
Confounder-dependent error	0.000	0.002	0.002	0.004	0.011	0.017
Covariate-dependent error	0.006	0.007	0.007	0.008	0.011	0.015

Fig. S12 (Simulation 2, censoring depending on (x_1, x_2, z), low percentage data removal)						
No delayed error	-0.010	-0.010	-0.009	-0.008	-0.004	-0.000
Independent error	-0.012	-0.010	-0.009	-0.008	-0.005	-0.001
Exposure-dependent error	-0.010	-0.008	-0.007	-0.005	-0.001	0.003
Confounder-dependent error	-0.011	-0.009	-0.007	-0.006	-0.002	0.002
Covariate-dependent error	-0.005	-0.003	-0.001	0.001	0.004	0.009
Fig. S13 (Simulation 2, censoring depending on (x_1, x_2, z), large percentage data removal)						
No delayed error	-0.004	-0.003	-0.003	-0.002	-0.000	0.002
Independent error	-0.005	-0.003	-0.002	0.001	0.007	0.017
Exposure-dependent error	0.000	0.003	0.004	0.005	0.009	0.017
Confounder-dependent error	0.000	0.003	0.003	0.004	0.009	0.017
Covariate-dependent error	0.005	0.007	0.008	0.008	0.010	0.016

Table S2: Bias of β coefficient for z from Model 1 (Cox) that corresponds to the simulations shown in Figures 1-13 in the supplement. Bias is presented for log(1) for log(1.1), log(1.15), log(1.25), log(1.5), log(2).

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	12	-	-
LRM_{obs}	0	0% (0/12)	100% (795838/795838)
LRM_u	4	33.33% (4/12)	100% (795838/795838)
LRM_{rl}	0	0% (0/12)	100% (795838/795838)
Delayed event time depends on significant SNPs			
Cox	19	100% (12/12)	99.9991% (795831/795838)
LRM_{obs}	15	100% (12/12)	99.9996% (795835/795838)
LRM_u	17	100% (12/12)	99.9994% (795833/795838)
LRM_{rl}	14	91.67% (11/12)	99.9996% (795835/795838)
Delayed event time depends on non-significant SNPs			
Cox	4	33.33% (4/12)	100% (795838/795838)
LRM_{obs}	0	0% (0/12)	100% (795838/795838)
LRM_u	1	8.33% (1/12)	100% (795838/795838)
LRM_{rl}	0	0% (0/12)	100% (795838/795838)
Delayed event time is independent			
Cox	0	0% (0/12)	100% (795838/795838)
LRM_{obs}	0	0% (0/12)	100% (795838/795838)
LRM_u	0	0% (0/12)	100% (795838/795838)
LRM_{rl}	0	0% (0/12)	100% (795838/795838)
Delayed event time depends on sex			
Cox	0	0% (0/12)	100% (795838/795838)
LRM_{obs}	0	0% (0/12)	100% (795838/795838)
LRM_u	0	0% (0/12)	100% (795838/795838)
LRM_{rl}	0	0% (0/12)	100% (795838/795838)

Table S3: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for cancer of bronchus; lung (phecode 165.1). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	29	-	-
LRM_{obs}	22	68.97% (20/29)	99.9997% (795819/795821)
LRM_{u}	24	79.31% (23/29)	99.9999% (795820/795821)
LRM_{rl}	20	62.07% (18/29)	99.9997% (795819/795821)
Delayed event time depends on significant SNPs			
Cox	54	55.17% (16/29)	99.9952% (795783/795821)
LRM_{obs}	52	51.72% (15/29)	99.9954% (795784/795821)
LRM_{u}	51	51.72% (15/29)	99.9955% (795785/795821)
LRM_{rl}	46	48.28% (14/29)	99.996% (795789/795821)
Delayed event time depends on non-significant SNPs			
Cox	23	51.72% (15/29)	99.999% (795813/795821)
LRM_{obs}	19	41.38% (12/29)	99.9991% (795814/795821)
LRM_{u}	17	41.38% (12/29)	99.9994% (795816/795821)
LRM_{rl}	20	41.38% (12/29)	99.999% (795813/795821)
Delayed event time is independent			
Cox	20	48.28% (14/29)	99.9992% (795815/795821)
LRM_{obs}	13	13.79% (4/29)	99.9989% (795812/795821)
LRM_{u}	15	27.59% (8/29)	99.9991% (795814/795821)
LRM_{rl}	14	20.69% (6/29)	99.999% (795813/795821)
Delayed event time depends on sex			
Cox	21	44.83% (13/29)	99.999% (795813/795821)
LRM_{obs}	19	13.79% (4/29)	99.9981% (795806/795821)
LRM_{u}	17	17.24% (5/29)	99.9985% (795809/795821)
LRM_{rl}	14	13.79% (4/29)	99.9987% (795811/795821)

Table S4: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for cancer of bronchus; lung (phecode 165.1). The results are shown for both the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	8	-	-
LRM_{obs}	3	37.5% (3/8)	100% (795842/795842)
LRM_{u}	0	0% (0/8)	100% (795842/795842)
LRM_{rl}	5	62.5% (5/8)	100% (795842/795842)
Delayed event time depends on significant SNPs			
Cox	6	75% (6/8)	100% (795842/795842)
LRM_{obs}	3	37.5% (3/8)	100% (795842/795842)
LRM_{u}	0	0% (0/8)	100% (795842/795842)
LRM_{rl}	3	37.5% (3/8)	100% (795842/795842)
Delayed event time depends on non-significant SNPs			
Cox	5	62.5% (5/8)	100% (795842/795842)
LRM_{obs}	3	37.5% (3/8)	100% (795842/795842)
LRM_{u}	0	0% (0/8)	100% (795842/795842)
LRM_{rl}	4	50% (4/8)	100% (795842/795842)
Delayed event time is independent			
Cox	0	0% (0/8)	100% (795842/795842)
LRM_{obs}	0	0% (0/8)	100% (795842/795842)
LRM_{u}	0	0% (0/8)	100% (795842/795842)
LRM_{rl}	0	0% (0/8)	100% (795842/795842)

Table S5: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for cancer of prostate (phecode 185). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	37	-	-
LRM_{obs}	32	51.35% (19/37)	99.9984% (795800/795813)
LRM_{u}	24	56.76% (21/37)	99.9996% (795810/795813)
LRM_{rl}	30	64.86% (24/37)	99.9992% (795807/795813)
Delayed event time depends on significant SNPs			
Cox	31	75.68% (28/37)	99.9996% (795810/795813)
LRM_{obs}	27	48.65% (18/37)	99.9989% (795804/795813)
LRM_{u}	24	51.35% (19/37)	99.9994% (795808/795813)
LRM_{rl}	28	64.86% (24/37)	99.9995% (795809/795813)
Delayed event time depends on non-significant SNPs			
Cox	34	67.57% (25/37)	99.9989% (795804/795813)
LRM_{obs}	32	40.54% (15/37)	99.9979% (795796/795813)
LRM_{u}	32	51.35% (19/37)	99.9984% (795800/795813)
LRM_{rl}	24	51.35% (19/37)	99.9994% (795808/795813)
Delayed event time is independent			
Cox	29	62.16% (23/37)	99.9992% (795807/795813)
LRM_{obs}	30	35.14% (13/37)	99.9979% (795796/795813)
LRM_{u}	28	43.24% (16/37)	99.9985% (795801/795813)
LRM_{rl}	23	48.65% (18/37)	99.9994% (795808/795813)

Table S6: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for cancer of prostate (phecode 185). The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox)- no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	231	-	-
LRM_{obs}	106	44.59% (103/231)	99.9996% (795616/795619)
LRM_{u}	126	54.11% (125/231)	99.9999% (795618/795619)
LRM_{rl}	194	83.55% (193/231)	99.9999% (795618/795619)
Delayed event time depends on significant SNPs			
Cox	239	92.21% (213/231)	99.9967% (795593/795619)
LRM_{obs}	124	48.48% (112/231)	99.9985% (795607/795619)
LRM_{u}	145	57.14% (132/231)	99.9984% (795606/795619)
LRM_{rl}	207	84.42% (195/231)	99.9985% (795607/795619)
Delayed event time depends on non-significant SNPs			
Cox	233	94.37% (218/231)	99.9981% (795604/795619)
LRM_{obs}	116	48.92% (113/231)	99.9996% (795616/795619)
LRM_{u}	140	56.71% (131/231)	99.9989% (795610/795619)
LRM_{rl}	210	87.45% (202/231)	99.999% (795611/795619)
Delayed event time is independent			
Cox	225	88.74% (205/231)	99.9975% (795599/795619)
LRM_{obs}	133	52.81% (122/231)	99.9986% (795608/795619)
LRM_{u}	142	57.14% (132/231)	99.9987% (795609/795619)
LRM_{rl}	162	66.67% (154/231)	99.999% (795611/795619)
Delayed event time depends on sex			
Cox	268	95.67% (221/231)	99.9941% (795572/795619)
LRM_{obs}	142	53.68% (124/231)	99.9977% (795601/795619)
LRM_{u}	162	58.87% (136/231)	99.9967% (795593/795619)
LRM_{rl}	235	91.34% (211/231)	99.997% (795595/795619)

Table S7: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for hypothyroidism (phecode 244). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	731	-	-
LRM_{obs}	434	56.22% (411/731)	99.9971% (795096/795119)
LRM_{u}	491	66.21% (484/731)	99.9991% (795112/795119)
LRM_{rl}	622	84.4% (617/731)	99.9994% (795114/795119)
Delayed event time depends on significant SNPs			
Cox	742	91.11% (666/731)	99.9904% (795043/795119)
LRM_{obs}	464	57.73% (422/731)	99.9947% (795077/795119)
LRM_{u}	540	64.98% (475/731)	99.9918% (795054/795119)
LRM_{rl}	644	79.62% (582/731)	99.9922% (795057/795119)
Delayed event time depends on non-significant SNPs			
Cox	753	92.48% (676/731)	99.9903% (795042/795119)
LRM_{obs}	446	55.4% (405/731)	99.9948% (795078/795119)
LRM_{u}	527	65.53% (479/731)	99.994% (795071/795119)
LRM_{rl}	637	79.62% (582/731)	99.9931% (795064/795119)
Delayed event time is independent			
Cox	731	88.92% (650/731)	99.9898% (795038/795119)
LRM_{obs}	488	60.33% (441/731)	99.9941% (795072/795119)
LRM_{u}	587	69.63% (509/731)	99.9902% (795041/795119)
LRM_{rl}	671	82.49% (603/731)	99.9914% (795051/795119)
Delayed event time depends on sex			
Cox	779	91.79% (671/731)	99.9864% (795011/795119)
LRM_{obs}	524	62.65% (458/731)	99.9917% (795053/795119)
LRM_{u}	617	71.82% (525/731)	99.9884% (795027/795119)
LRM_{rl}	725	86.87% (635/731)	99.9887% (795029/795119)

Table S8: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for hypothyroidism (phecode 244). The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	298	-	-
LRM_{obs}	153	48.66% (145/298)	99.999% (795544/795552)
LRM_{u}	196	61.74% (184/298)	99.9985% (795540/795552)
LRM_{rl}	268	88.93% (265/298)	99.9996% (795549/795552)
Delayed event time depends on significant SNPs			
Cox	201	66.11% (197/298)	99.9995% (795548/795552)
LRM_{obs}	129	41.61% (124/298)	99.9994% (795547/795552)
LRM_{u}	164	53.69% (160/298)	99.9995% (795548/795552)
LRM_{rl}	168	55.37% (165/298)	99.9996% (795549/795552)
Delayed event time depends on non-significant SNPs			
Cox	213	69.8% (208/298)	99.9994% (795547/795552)
LRM_{obs}	124	38.59% (115/298)	99.9989% (795543/795552)
LRM_{u}	165	53.36% (159/298)	99.9992% (795546/795552)
LRM_{rl}	206	67.45% (201/298)	99.9994% (795547/795552)
Delayed event time is independent			
Cox	224	71.81% (214/298)	99.9987% (795542/795552)
LRM_{obs}	148	47.32% (141/298)	99.9991% (795545/795552)
LRM_{u}	183	59.06% (176/298)	99.9991% (795545/795552)
LRM_{rl}	214	69.13% (206/298)	99.999% (795544/795552)
Delayed event time depends on sex			
Cox	258	80.54% (240/298)	99.9977% (795534/795552)
LRM_{obs}	193	62.42% (186/298)	99.9991% (795545/795552)
LRM_{u}	211	67.11% (200/298)	99.9986% (795541/795552)
LRM_{rl}	250	76.85% (229/298)	99.9974% (795531/795552)

Table S9: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for type 2 diabetes (phecode 250.2). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox)- no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	548	-	-
LRM_{obs}	483	76.28% (418/548)	99.9918% (795237/795302)
LRM_{u}	474	79.74% (437/548)	99.9953% (795265/795302)
LRM_{rl}	526	89.05% (488/548)	99.9952% (795264/795302)
Delayed event time depends on significant SNPs			
Cox	549	89.05% (488/548)	99.9923% (795241/795302)
LRM_{obs}	433	70.07% (384/548)	99.9938% (795253/795302)
LRM_{u}	454	78.28% (429/548)	99.9969% (795277/795302)
LRM_{rl}	535	85.4% (468/548)	99.9916% (795235/795302)
Delayed event time depends on non-significant SNPs			
Cox	578	92.34% (506/548)	99.9909% (795230/795302)
LRM_{obs}	465	68.43% (375/548)	99.9887% (795212/795302)
LRM_{u}	504	79.56% (436/548)	99.9914% (795234/795302)
LRM_{rl}	581	86.5% (474/548)	99.9865% (795195/795302)
Delayed event time is independent			
Cox	616	90.51% (496/548)	99.9849% (795182/795302)
LRM_{obs}	485	72.08% (395/548)	99.9887% (795212/795302)
LRM_{u}	534	79.93% (438/548)	99.9879% (795206/795302)
LRM_{rl}	606	86.68% (475/548)	99.9835% (795171/795302)
Delayed event time depends on sex			
Cox	603	88.69% (486/548)	99.9853% (795185/795302)
LRM_{obs}	490	69.34% (380/548)	99.9862% (795192/795302)
LRM_{u}	529	79.2% (434/548)	99.9881% (795207/795302)
LRM_{rl}	581	86.5% (474/548)	99.9865% (795195/795302)

Table S10: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for type 2 diabetes (phecode 250.2). The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	3	-	-
LRM_{obs}	3	100% (3/3)	100% (795847/795847)
LRM_{u}	3	100% (3/3)	100% (795847/795847)
LRM_{rl}	3	100% (3/3)	100% (795847/795847)
Delayed event time depends on significant SNPs			
Cox	5	66.67% (2/3)	99.9996% (795844/795847)
LRM_{obs}	5	66.67% (2/3)	99.9996% (795844/795847)
LRM_{u}	5	66.67% (2/3)	99.9996% (795844/795847)
LRM_{rl}	5	66.67% (2/3)	99.9996% (795844/795847)
Delayed event time depends on non-significant SNPs			
Cox	6	100% (3/3)	99.9996% (795844/795847)
LRM_{obs}	6	100% (3/3)	99.9996% (795844/795847)
LRM_{u}	6	100% (3/3)	99.9996% (795844/795847)
LRM_{rl}	6	100% (3/3)	99.9996% (795844/795847)
Delayed event time is independent			
Cox	6	100% (3/3)	99.9996% (795844/795847)
LRM_{obs}	7	100% (3/3)	99.9995% (795843/795847)
LRM_{u}	7	100% (3/3)	99.9995% (795843/795847)
LRM_{rl}	6	100% (3/3)	99.9996% (795844/795847)
Delayed event time depends on sex			
Cox	6	100% (3/3)	99.9996% (795844/795847)
LRM_{obs}	7	100% (3/3)	99.9995% (795843/795847)
LRM_{u}	7	100% (3/3)	99.9995% (795843/795847)
LRM_{rl}	6	100% (3/3)	99.9996% (795844/795847)

Table S11: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for vitamin D deficiency (phecode 261.4). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	13	-	-
LRM_{obs}	16	92.31% (12/13)	99.9995% (795833/795837)
LRM_{u}	19	92.31% (12/13)	99.9991% (795830/795837)
LRM_{rl}	18	100% (13/13)	99.9994% (795832/795837)
Delayed event time depends on significant SNPs			
Cox	14	76.92% (10/13)	99.9995% (795833/795837)
LRM_{obs}	23	76.92% (10/13)	99.9984% (795824/795837)
LRM_{u}	22	76.92% (10/13)	99.9985% (795825/795837)
LRM_{rl}	18	84.62% (11/13)	99.9991% (795830/795837)
Delayed event time depends on non-significant SNPs			
Cox	17	84.62% (11/13)	99.9992% (795831/795837)
LRM_{obs}	21	84.62% (11/13)	99.9987% (795827/795837)
LRM_{u}	22	84.62% (11/13)	99.9986% (795826/795837)
LRM_{rl}	19	84.62% (11/13)	99.999% (795829/795837)
Delayed event time is independent			
Cox	20	84.62% (11/13)	99.9989% (795828/795837)
LRM_{obs}	21	84.62% (11/13)	99.9987% (795827/795837)
LRM_{u}	21	84.62% (11/13)	99.9987% (795827/795837)
LRM_{rl}	21	84.62% (11/13)	99.9987% (795827/795837)
Delayed event time depends on sex			
Cox	17	84.62% (11/13)	99.9992% (795831/795837)
LRM_{obs}	23	84.62% (11/13)	99.9985% (795825/795837)
LRM_{u}	22	84.62% (11/13)	99.9986% (795826/795837)
LRM_{rl}	21	92.31% (12/13)	99.9989% (795828/795837)

Table S12: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for vitamin D deficiency (phecode 261.4). The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	15	-	-
LRM_{obs}	16	80% (12/15)	99.9995% (795831/795835)
LRM_u	16	80% (12/15)	99.9995% (795831/795835)
LRM_{rl}	15	100% (15/15)	100% (795835/795835)
Delayed event time depends on significant SNPs			
Cox	11	73.33% (11/15)	100% (795835/795835)
LRM_{obs}	13	53.33% (8/15)	99.9994% (795830/795835)
LRM_u	14	73.33% (11/15)	99.9996% (795832/795835)
LRM_{rl}	11	73.33% (11/15)	100% (795835/795835)
Delayed event time depends on non-significant SNPs			
Cox	14	93.33% (14/15)	100% (795835/795835)
LRM_{obs}	16	73.33% (11/15)	99.9994% (795830/795835)
LRM_u	14	73.33% (11/15)	99.9996% (795832/795835)
LRM_{rl}	15	100% (15/15)	100% (795835/795835)
Delayed event time is independent			
Cox	13	86.67% (13/15)	100% (795835/795835)
LRM_{obs}	16	73.33% (11/15)	99.9994% (795830/795835)
LRM_u	14	73.33% (11/15)	99.9996% (795832/795835)
LRM_{rl}	15	100% (15/15)	100% (795835/795835)
Delayed event time depends on sex			
Cox	14	93.33% (14/15)	100% (795835/795835)
LRM_{obs}	16	73.33% (11/15)	99.9994% (795830/795835)
LRM_u	14	73.33% (11/15)	99.9996% (795832/795835)
LRM_{rl}	14	93.33% (14/15)	100% (795835/795835)

Table S13: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for hypercholesterolemia (phecode 272.11). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	60	-	-
LRM_{obs}	34	41.67% (25/60)	99.9989% (795781/795790)
LRM_{u}	44	60% (36/60)	99.999% (795782/795790)
LRM_{rl}	52	85% (51/60)	99.9999% (795789/795790)
Delayed event time depends on significant SNPs			
Cox	53	83.33% (50/60)	99.9996% (795787/795790)
LRM_{obs}	43	45% (27/60)	99.998% (795774/795790)
LRM_{u}	41	53.33% (32/60)	99.9989% (795781/795790)
LRM_{rl}	53	80% (48/60)	99.9994% (795785/795790)
Delayed event time depends on non-significant SNPs			
Cox	58	93.33% (56/60)	99.9997% (795788/795790)
LRM_{obs}	37	40% (24/60)	99.9984% (795777/795790)
LRM_{u}	45	58.33% (35/60)	99.9987% (795780/795790)
LRM_{rl}	55	86.67% (52/60)	99.9996% (795787/795790)
Delayed event time is independent			
Cox	45	68.33% (41/60)	99.9995% (795786/795790)
LRM_{obs}	44	46.67% (28/60)	99.998% (795774/795790)
LRM_{u}	44	53.33% (32/60)	99.9985% (795778/795790)
LRM_{rl}	45	66.67% (40/60)	99.9994% (795785/795790)
Delayed event time depends on sex			
Cox	58	83.33% (50/60)	99.999% (795782/795790)
LRM_{obs}	47	50% (30/60)	99.9979% (795773/795790)
LRM_{u}	51	55% (33/60)	99.9977% (795772/795790)
LRM_{rl}	56	78.33% (47/60)	99.9989% (795781/795790)

Table S14: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for hypercholesterolemia (phecode 272.11). The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	1	-	-
LRM_{obs}	0	0% (0/1)	100% (795849/795849)
LRM_{u}	0	0% (0/1)	100% (795849/795849)
LRM_{rl}	0	0% (0/1)	100% (795849/795849)
Delayed event time depends on significant SNPs			
Cox	1	100% (1/1)	100% (795849/795849)
LRM_{obs}	1	100% (1/1)	100% (795849/795849)
LRM_{u}	1	100% (1/1)	100% (795849/795849)
LRM_{rl}	1	100% (1/1)	100% (795849/795849)
Delayed event time depends on non-significant SNPs			
Cox	0	0% (0/1)	100% (795849/795849)
LRM_{obs}	0	0% (0/1)	100% (795849/795849)
LRM_{u}	0	0% (0/1)	100% (795849/795849)
LRM_{rl}	0	0% (0/1)	100% (795849/795849)
Delayed event time is independent			
Cox	0	0% (0/1)	100% (795849/795849)
LRM_{obs}	0	0% (0/1)	100% (795849/795849)
LRM_{u}	0	0% (0/1)	100% (795849/795849)
LRM_{rl}	0	0% (0/1)	100% (795849/795849)
Delayed event time depends on sex			
Cox	0	0% (0/1)	100% (795849/795849)
LRM_{obs}	0	0% (0/1)	100% (795849/795849)
LRM_{u}	0	0% (0/1)	100% (795849/795849)
LRM_{rl}	0	0% (0/1)	100% (795849/795849)

Table S15: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for insomnia (phecode 327.4). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	12	-	-
LRM_{obs}	14	66.67% (8/12)	99.9992% (795832/795838)
LRM_{u}	15	83.33% (10/12)	99.9994% (795833/795838)
LRM_{rl}	13	100% (12/12)	99.9999% (795837/795838)
Delayed event time depends on significant SNPs			
Cox	13	58.33% (7/12)	99.9992% (795832/795838)
LRM_{obs}	13	50% (6/12)	99.9991% (795831/795838)
LRM_{u}	13	58.33% (7/12)	99.9992% (795832/795838)
LRM_{rl}	14	58.33% (7/12)	99.9991% (795831/795838)
Delayed event time depends on non-significant SNPs			
Cox	10	66.67% (8/12)	99.9997% (795836/795838)
LRM_{obs}	12	50% (6/12)	99.9992% (795832/795838)
LRM_{u}	13	66.67% (8/12)	99.9994% (795833/795838)
LRM_{rl}	13	75% (9/12)	99.9995% (795834/795838)
Delayed event time is independent			
Cox	9	50% (6/12)	99.9996% (795835/795838)
LRM_{obs}	10	33.33% (4/12)	99.9992% (795832/795838)
LRM_{u}	13	58.33% (7/12)	99.9992% (795832/795838)
LRM_{rl}	15	83.33% (10/12)	99.9994% (795833/795838)
Delayed event time depends on sex			
Cox	14	75% (9/12)	99.9994% (795833/795838)
LRM_{obs}	13	58.33% (7/12)	99.9992% (795832/795838)
LRM_{u}	16	66.67% (8/12)	99.999% (795830/795838)
LRM_{rl}	14	75% (9/12)	99.9994% (795833/795838)

Table S16: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for insomnia (phecode 327.4). The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	4	-	-
LRM_{obs}	6	75% (3/4)	99.9996% (795843/795846)
LRM_{u}	6	100% (4/4)	99.9997% (795844/795846)
LRM_{rl}	3	75% (3/4)	100% (795846/795846)
Delayed event time depends on significant SNPs			
Cox	4	100% (4/4)	100% (795846/795846)
LRM_{obs}	6	100% (4/4)	99.9997% (795844/795846)
LRM_{u}	6	100% (4/4)	99.9997% (795844/795846)
LRM_{rl}	4	100% (4/4)	100% (795846/795846)
Delayed event time depends on non-significant SNPs			
Cox	3	75% (3/4)	100% (795846/795846)
LRM_{obs}	5	75% (3/4)	99.9997% (795844/795846)
LRM_{u}	6	100% (4/4)	99.9997% (795844/795846)
LRM_{rl}	3	75% (3/4)	100% (795846/795846)
Delayed event time is independent			
Cox	3	75% (3/4)	100% (795846/795846)
LRM_{obs}	4	50% (2/4)	99.9997% (795844/795846)
LRM_{u}	6	100% (4/4)	99.9997% (795844/795846)
LRM_{rl}	3	75% (3/4)	100% (795846/795846)
Delayed event time depends on sex			
Cox	1	25% (1/4)	100% (795846/795846)
LRM_{obs}	3	25% (1/4)	99.9997% (795844/795846)
LRM_{u}	4	50% (2/4)	99.9997% (795844/795846)
LRM_{rl}	1	25% (1/4)	100% (795846/795846)

Table S17: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for myocardial infarction (phecode 411.2). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	19	-	-
LRM_{obs}	29	57.89% (11/19)	99.9977% (795813/795831)
LRM_{u}	24	68.42% (13/19)	99.9986% (795820/795831)
LRM_{rl}	17	78.95% (15/19)	99.9997% (795829/795831)
Delayed event time depends on significant SNPs			
Cox	17	36.84% (7/19)	99.9987% (795821/795831)
LRM_{obs}	23	42.11% (8/19)	99.9981% (795816/795831)
LRM_{u}	17	36.84% (7/19)	99.9987% (795821/795831)
LRM_{rl}	19	42.11% (8/19)	99.9986% (795820/795831)
Delayed event time depends on non-significant SNPs			
Cox	21	52.63% (10/19)	99.9986% (795820/795831)
LRM_{obs}	23	36.84% (7/19)	99.998% (795815/795831)
LRM_{u}	24	47.37% (9/19)	99.9981% (795816/795831)
LRM_{rl}	17	47.37% (9/19)	99.999% (795823/795831)
Delayed event time is independent			
Cox	18	31.58% (6/19)	99.9985% (795819/795831)
LRM_{obs}	26	36.84% (7/19)	99.9976% (795812/795831)
LRM_{u}	17	36.84% (7/19)	99.9987% (795821/795831)
LRM_{rl}	17	42.11% (8/19)	99.9989% (795822/795831)
Delayed event time depends on sex			
Cox	19	42.11% (8/19)	99.9986% (795820/795831)
LRM_{obs}	20	31.58% (6/19)	99.9982% (795817/795831)
LRM_{u}	19	36.84% (7/19)	99.9985% (795819/795831)
LRM_{rl}	15	36.84% (7/19)	99.999% (795823/795831)

Table S18: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for myocardial infarction (phecode 411.2). The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	181	-	-
LRM_{obs}	75	27.62% (50/181)	99.9969% (795644/795669)
LRM_{u}	117	51.93% (94/181)	99.9971% (795646/795669)
LRM_{rl}	164	88.95% (161/181)	99.9996% (795666/795669)
Delayed event time depends on significant SNPs			
Cox	152	81.77% (148/181)	99.9995% (795665/795669)
LRM_{obs}	68	27.07% (49/181)	99.9976% (795650/795669)
LRM_{u}	83	34.25% (62/181)	99.9974% (795648/795669)
LRM_{rl}	87	46.41% (84/181)	99.9996% (795666/795669)
Delayed event time depends on non-significant SNPs			
Cox	165	91.16% (165/181)	100% (795669/795669)
LRM_{obs}	67	27.07% (49/181)	99.9977% (795651/795669)
LRM_{u}	78	32.6% (59/181)	99.9976% (795650/795669)
LRM_{rl}	92	50.28% (91/181)	99.9999% (795668/795669)
Delayed event time is independent			
Cox	164	90.61% (164/181)	100% (795669/795669)
LRM_{obs}	73	30.94% (56/181)	99.9979% (795652/795669)
LRM_{u}	112	53.04% (96/181)	99.998% (795653/795669)
LRM_{rl}	122	66.3% (120/181)	99.9997% (795667/795669)
Delayed event time depends on sex			
Cox	139	76.24% (138/181)	99.9999% (795668/795669)
LRM_{obs}	70	28.73% (52/181)	99.9977% (795651/795669)
LRM_{u}	88	38.67% (70/181)	99.9977% (795651/795669)
LRM_{rl}	85	46.96% (85/181)	100% (795669/795669)

Table S19: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for coronary atherosclerosis (phecode 411.4). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	423	-	-
LRM_{obs}	234	39.48% (167/423)	99.9916% (795360/795427)
LRM_{u}	275	51.77% (219/423)	99.993% (795371/795427)
LRM_{rl}	234	52.48% (222/423)	99.9985% (795415/795427)
Delayed event time depends on significant SNPs			
Cox	394	79.43% (336/423)	99.9927% (795369/795427)
LRM_{obs}	181	31.21% (132/423)	99.9938% (795378/795427)
LRM_{u}	246	48.23% (204/423)	99.9947% (795385/795427)
LRM_{rl}	220	50.12% (212/423)	99.999% (795419/795427)
Delayed event time depends on non-significant SNPs			
Cox	365	74.7% (316/423)	99.9938% (795378/795427)
LRM_{obs}	178	30.02% (127/423)	99.9936% (795376/795427)
LRM_{u}	248	47.04% (199/423)	99.9938% (795378/795427)
LRM_{rl}	214	48.94% (207/423)	99.9991% (795420/795427)
Delayed event time is independent			
Cox	343	72.34% (306/423)	99.9953% (795390/795427)
LRM_{obs}	220	40.9% (173/423)	99.9941% (795380/795427)
LRM_{u}	246	48.7% (206/423)	99.995% (795387/795427)
LRM_{rl}	216	49.65% (210/423)	99.9992% (795421/795427)
Delayed event time depends on sex			
Cox	325	69.03% (292/423)	99.9959% (795394/795427)
LRM_{obs}	195	33.57% (142/423)	99.9933% (795374/795427)
LRM_{u}	247	48.7% (206/423)	99.9948% (795386/795427)
LRM_{rl}	212	48.23% (204/423)	99.999% (795419/795427)

Table S20: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for coronary atherosclerosis (phecode 411.4). The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	126	-	-
LRM_{obs}	126	92.86% (117/126)	99.9989% (795715/795724)
LRM_{u}	123	93.65% (118/126)	99.9994% (795719/795724)
LRM_{rl}	123	94.44% (119/126)	99.9995% (795720/795724)
Delayed event time depends on significant SNPs			
Cox	118	93.65% (118/126)	100% (795724/795724)
LRM_{obs}	121	92.06% (116/126)	99.9994% (795719/795724)
LRM_{u}	121	92.06% (116/126)	99.9994% (795719/795724)
LRM_{rl}	116	91.27% (115/126)	99.9999% (795723/795724)
Delayed event time depends on non-significant SNPs			
Cox	122	96.83% (122/126)	100% (795724/795724)
LRM_{obs}	121	91.27% (115/126)	99.9992% (795718/795724)
LRM_{u}	121	92.86% (117/126)	99.9995% (795720/795724)
LRM_{rl}	116	91.27% (115/126)	99.9999% (795723/795724)
Delayed event time is independent			
Cox	118	93.65% (118/126)	100% (795724/795724)
LRM_{obs}	119	91.27% (115/126)	99.9995% (795720/795724)
LRM_{u}	121	92.86% (117/126)	99.9995% (795720/795724)
LRM_{rl}	115	91.27% (115/126)	100% (795724/795724)
Delayed event time depends on sex			
Cox	116	92.06% (116/126)	100% (795724/795724)
LRM_{obs}	119	91.27% (115/126)	99.9995% (795720/795724)
LRM_{u}	121	92.86% (117/126)	99.9995% (795720/795724)
LRM_{rl}	114	90.48% (114/126)	100% (795724/795724)

Table S21: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for atrial fibrillation (phecode 427.21). The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	186	-	-
LRM_{obs}	197	72.04% (134/186)	99.9921% (795601/795664)
LRM_{u}	194	75.81% (141/186)	99.9933% (795611/795664)
LRM_{rl}	186	83.87% (156/186)	99.9962% (795634/795664)
Delayed event time depends on significant SNPs			
Cox	175	83.33% (155/186)	99.9975% (795644/795664)
LRM_{obs}	177	69.89% (130/186)	99.9941% (795617/795664)
LRM_{u}	173	70.97% (132/186)	99.9948% (795623/795664)
LRM_{rl}	156	76.88% (143/186)	99.9984% (795651/795664)
Delayed event time depends on non-significant SNPs			
Cox	176	83.33% (155/186)	99.9974% (795643/795664)
LRM_{obs}	151	69.89% (130/186)	99.9974% (795643/795664)
LRM_{u}	149	72.04% (134/186)	99.9981% (795649/795664)
LRM_{rl}	158	76.34% (142/186)	99.998% (795648/795664)
Delayed event time is independent			
Cox	179	83.33% (155/186)	99.997% (795640/795664)
LRM_{obs}	157	70.43% (131/186)	99.9967% (795638/795664)
LRM_{u}	161	72.58% (135/186)	99.9967% (795638/795664)
LRM_{rl}	163	77.96% (145/186)	99.9977% (795646/795664)
Delayed event time depends on sex			
Cox	165	81.18% (151/186)	99.9982% (795650/795664)
LRM_{obs}	155	70.43% (131/186)	99.997% (795640/795664)
LRM_{u}	147	70.43% (131/186)	99.998% (795648/795664)
LRM_{rl}	154	75.81% (141/186)	99.9984% (795651/795664)

Table S22: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for atrial fibrillation (phecode 427.21). The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 5 \times 10^{-8}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	879	-	-
LRM_{obs}	488	49.6% (436/879)	99.9993% (7957569/7957621)
LRM_u	591	61.89% (544/879)	99.9994% (7957574/7957621)
LRM_{rl}	775	86.92% (764/879)	99.9999% (7957610/7957621)
Delayed event time depends on significant SNPs			
Cox	756	81% (712/879)	99.9994% (7957577/7957621)
LRM_{obs}	485	49.03% (431/879)	99.9993% (7957567/7957621)
LRM_u	556	56.88% (500/879)	99.9993% (7957565/7957621)
LRM_{rl}	616	67.24% (591/879)	99.9997% (7957596/7957621)
Delayed event time depends on non-significant SNPs			
Cox	765	84.41% (742/879)	99.9997% (7957598/7957621)
LRM_{obs}	458	46.87% (412/879)	99.9994% (7957575/7957621)
LRM_u	531	55.18% (485/879)	99.9994% (7957575/7957621)
LRM_{rl}	652	72.13% (634/879)	99.9998% (7957603/7957621)
Delayed event time is independent			
Cox	753	81.91% (720/879)	99.9996% (7957588/7957621)
LRM_{obs}	500	51.19% (450/879)	99.9994% (7957571/7957621)
LRM_u	585	61.32% (539/879)	99.9994% (7957575/7957621)
LRM_{rl}	637	70.08% (616/879)	99.9997% (7957600/7957621)
Delayed event time depends on sex			
Cox	802	84.16% (733/871)	99.999% (7161710/7161779)
LRM_{obs}	550	56.49% (492/871)	99.9992% (7161721/7161779)
LRM_u	607	61.88% (539/871)	99.9991% (7161711/7161779)
LRM_{rl}	705	75.43% (657/871)	99.9993% (7161731/7161779)

Table S23: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for all ten phecodes. The results are shown for the $P \leq 5 \times 10^{-8}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

$P \leq 1 \times 10^{-5}$	# Significant SNPs	True Positive *	True Negative *
No delayed event time			
Cox	2058	-	-
LRM_{obs}	1495	59.52% (1225/2058)	99.9966% (7956172/7956442)
LRM_{u}	1584	67.83% (1396/2058)	99.9976% (7956254/7956442)
LRM_{rl}	1718	78.52% (1616/2058)	99.9987% (7956340/7956442)
Delayed event time depends on significant SNPs			
Cox	2042	85.67% (1763/2058)	99.9965% (7956163/7956442)
LRM_{obs}	1436	55.98% (1152/2058)	99.9964% (7956158/7956442)
LRM_{u}	1581	64.63% (1330/2058)	99.9968% (7956191/7956442)
LRM_{rl}	1733	73.71% (1517/2058)	99.9973% (7956226/7956442)
Delayed event time depends on non-significant SNPs			
Cox	2035	86.39% (1778/2058)	99.9968% (7956185/7956442)
LRM_{obs}	1384	54.03% (1112/2058)	99.9966% (7956170/7956442)
LRM_{u}	1581	65.21% (1342/2058)	99.997% (7956203/7956442)
LRM_{rl}	1738	73.71% (1517/2058)	99.9972% (7956221/7956442)
Delayed event time is independent			
Cox	2010	82.99% (1708/2058)	99.9962% (7956140/7956442)
LRM_{obs}	1494	58.65% (1207/2058)	99.9964% (7956155/7956442)
LRM_{u}	1666	66.52% (1369/2058)	99.9963% (7956145/7956442)
LRM_{rl}	1791	74.15% (1526/2058)	99.9967% (7956177/7956442)
Delayed event time depends on sex			
Cox	2001	83.67% (1691/2021)	99.9957% (7160319/7160629)
LRM_{obs}	1486	57.84% (1169/2021)	99.9956% (7160312/7160629)
LRM_{u}	1665	67.29% (1360/2021)	99.9957% (7160324/7160629)
LRM_{rl}	1792	75.85% (1533/2021)	99.9964% (7160370/7160629)

Table S24: Number of significant SNPs, true positive rates, and true negative rates for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for all ten phecodes. The results are shown for the $P \leq 1 \times 10^{-5}$ significance level.

* Based on Model 1 (Cox) - no delayed event time

	$P \leq 5 \times 10^{-8}$		$P \leq 1 \times 10^{-5}$	
	TPR (95% CI) *	**TNR (95% CI) ***	**TPR (95% CI) ***	**TNR (95% CI) ***
No delayed event time				
Cox	Reference	Reference	Reference	Reference
LRM_{obs}	50.62% (44.49%, 56.75%)	99.99% (99.99%, 100.00%)	62.29% (57.72%, 66.85%)	99.99% (99.97%, 100.00%)
LRM_{u}	57.48% (50.17%, 64.78%)	99.99% (99.99%, 100.00%)	71.37% (67.20%, 75.54%)	99.99% (99.98%, 100.00%)
LRM_{rl}	69.34% (62.73%, 75.94%)	100.00% (99.99%, 100.00%)	80.07% (75.34%, 84.79%)	99.99% (99.98%, 100.00%)
Delayed event time depends on significant SNPs				
Cox	84.87% (78.72%, 91.03%)	99.99% (99.95%, 100.00%)	72.92% (67.57%, 78.27%)	99.99% (99.93%, 100.00%)
LRM_{obs}	66.67% (60.31%, 73.03%)	99.99% (99.96%, 100.00%)	54.33% (49.08%, 59.58%)	99.99% (99.93%, 100.00%)
LRM_{u}	67.72% (62.31%, 73.12%)	99.99% (99.96%, 100.00%)	59.10% (53.80%, 64.39%)	99.99% (99.93%, 100.00%)
LRM_{rl}	74.66% (67.29%, 82.04%)	100.00% (99.97%, 100.00%)	67.02% (61.69%, 72.35%)	99.99% (99.94%, 100.00%)
Delayed event time depends on non-significant SNPs				
Cox	71.63% (63.44%, 79.82%)	100.00% (99.98%, 100.00%)	75.94% (70.58%, 81.30%)	99.99% (99.96%, 100.00%)
LRM_{obs}	49.17% (43.21%, 55.12%)	99.99% (99.98%, 100.00%)	51.71% (46.63%, 56.79%)	99.99% (99.96%, 100.00%)
LRM_{u}	51.72% (45.85%, 57.59%)	99.99% (99.98%, 100.00%)	61.40% (56.17%, 66.61%)	99.99% (99.96%, 100.00%)
LRM_{rl}	62.14% (55.48%, 68.81%)	100.00% (99.98%, 100.00%)	67.78% (62.36%, 73.20%)	99.99% (99.96%, 100.00%)
Delayed event time is independent				
Cox	60.65% (55.34%, 65.96%)	100.00% (99.98%, 100.00%)	68.01% (62.75%, 73.26%)	99.99% (99.96%, 100.00%)
LRM_{obs}	44.57% (39.65%, 49.49%)	99.99% (99.97%, 100.00%)	49.41% (44.98%, 53.85%)	99.99% (99.96%, 100.00%)
LRM_{u}	53.54% (48.62%, 58.46%)	99.99% (99.97%, 100.00%)	57.48% (52.48%, 62.48%)	99.99% (99.96%, 100.00%)
LRM_{rl}	56.84% (51.12%, 62.55%)	100.00% (99.98%, 100.00%)	64.28% (59.34%, 69.23%)	99.99% (99.96%, 100.00%)
Delayed event time depends on sex				
Cox	62.54% (57.05%, 68.03%)	99.99% (99.97%, 100.00%)	73.40% (68.26%, 78.54%)	99.99% (99.96%, 100.00%)
LRM_{obs}	48.27% (43.35%, 53.19%)	99.99% (99.97%, 100.00%)	52.70% (48.49%, 56.91%)	99.99% (99.95%, 100.00%)
LRM_{u}	53.43% (48.51%, 58.34%)	99.99% (99.97%, 100.00%)	58.95% (54.52%, 63.37%)	99.99% (99.95%, 100.00%)
LRM_{rl}	58.22% (52.72%, 63.72%)	99.99% (99.98%, 100.00%)	65.96% (61.60%, 70.33%)	99.99% (99.96%, 100.00%)

Table S25: Average true positive and true negative rates (95% confidence interval) for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard. The average is calculated from ten phecodes, which are given in Appendix B, Table 1. The results are shown for both the $P \leq 5 \times 10^{-8}$ and $P \leq 1 \times 10^{-5}$ significance levels.

* Based on Model 1 (Cox) - no delayed event time

3 Appendix C: Additional Figures

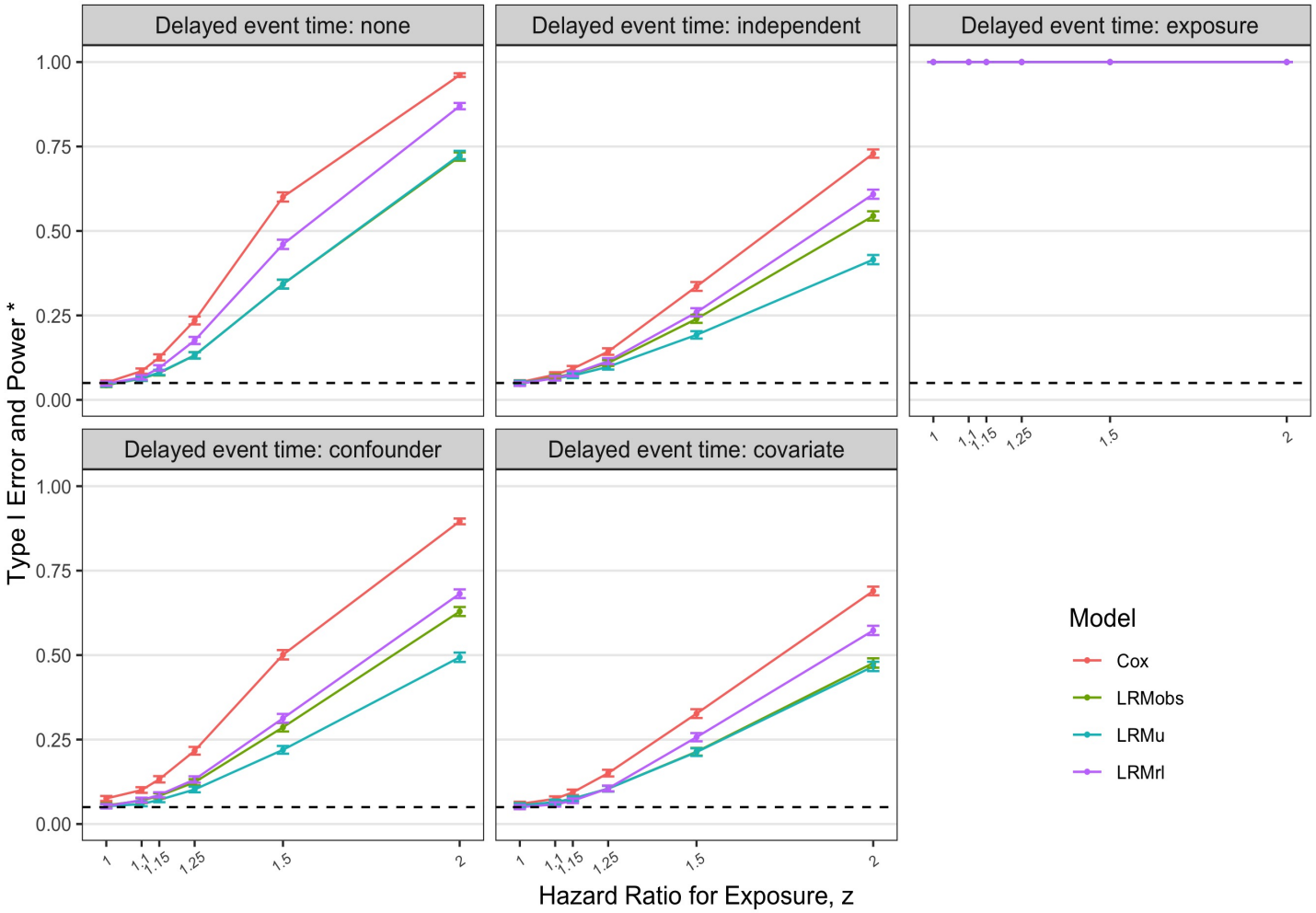

Fig. S1: Results from Simulation 1 (with truncation) when the event time was generated from a Cox model with baseline hazard from an exponential distribution, the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x, with left truncation, and the observations with simulated event times before truncation were removed from the analysis (*removal-practice*). The parameters led to a large number of observations with a misclassified event status (detailed in Supplementary A).

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

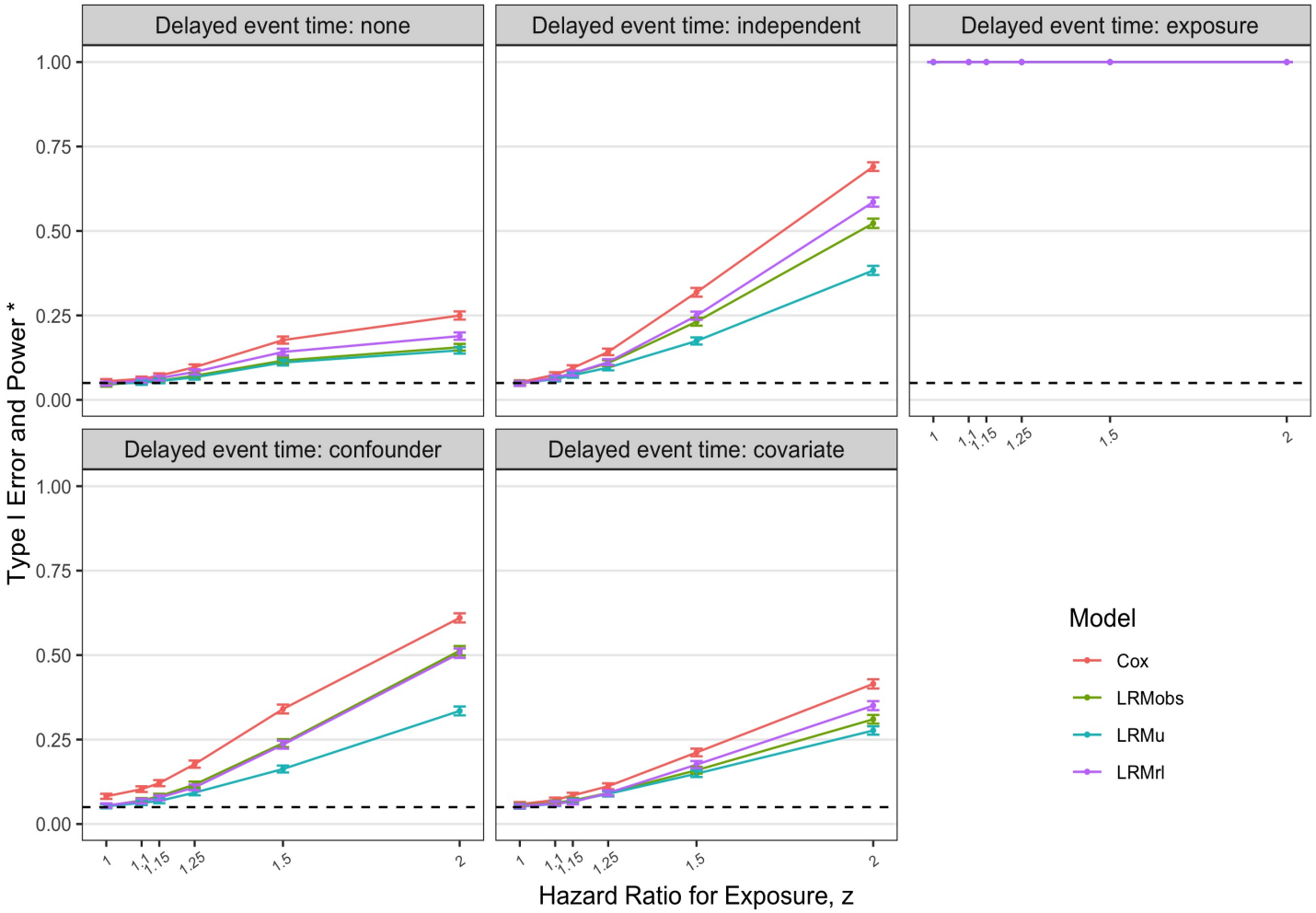

Fig. S2: Results from Simulation 1 (with truncation) when the event time was generated from a Cox model with baseline hazard from an exponential distribution, the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x, with left truncation, and the observations with simulated event times before truncation were considered as censored in the analysis (*censor-practice*). The parameters led to a large number of observations with a misclassified event status (detailed in Supplementary A).

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

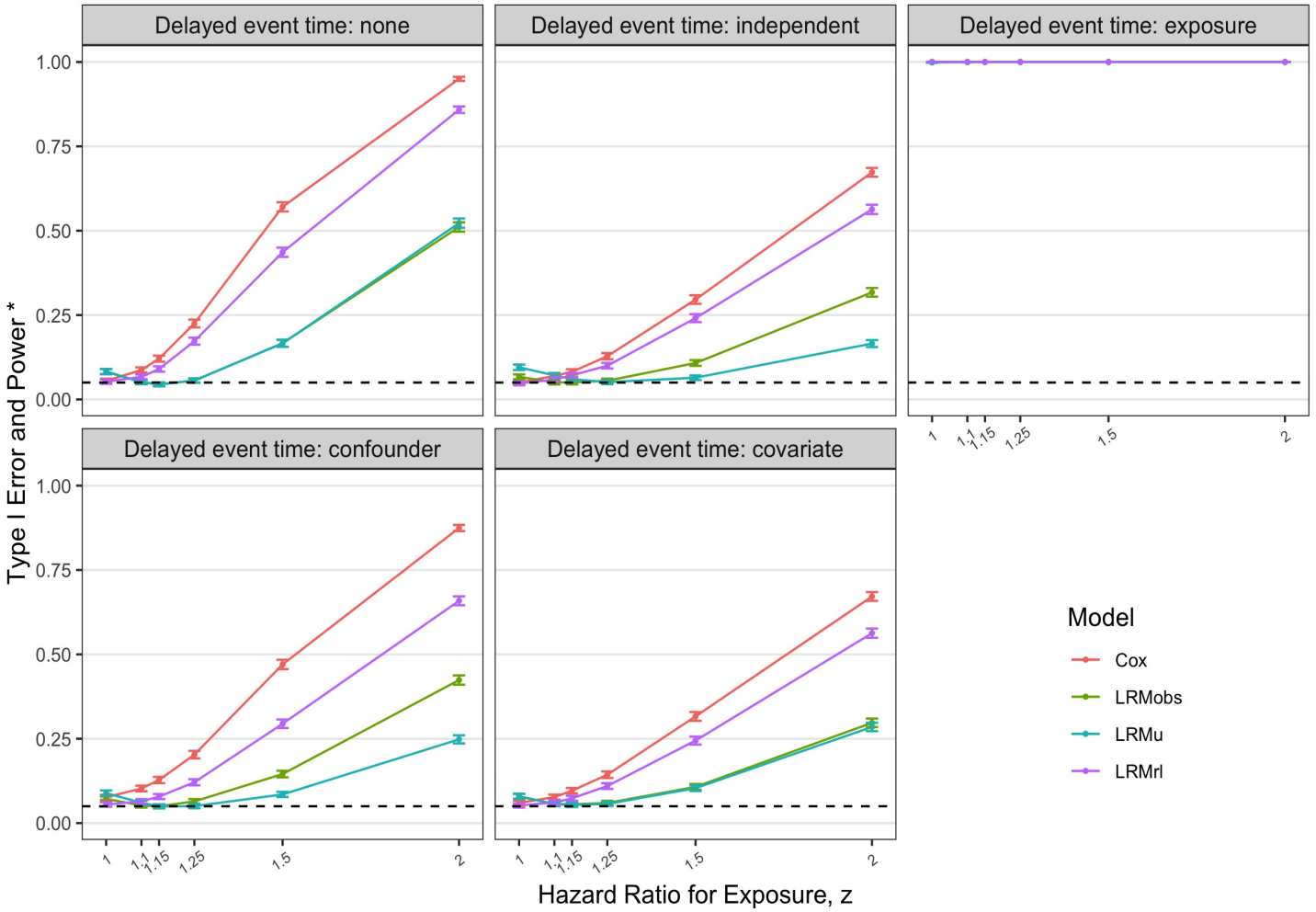

Fig. S3: Results from Simulation 1 (with truncation) when the event time was generated from a Cox model with baseline hazard from an exponential distribution, the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x and z, with left truncation, and the observations with simulated event times before truncation were removed from the analysis (*removal-practice*). The parameters led to a large number of observations with a misclassified event status (detailed in Appendix A).

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

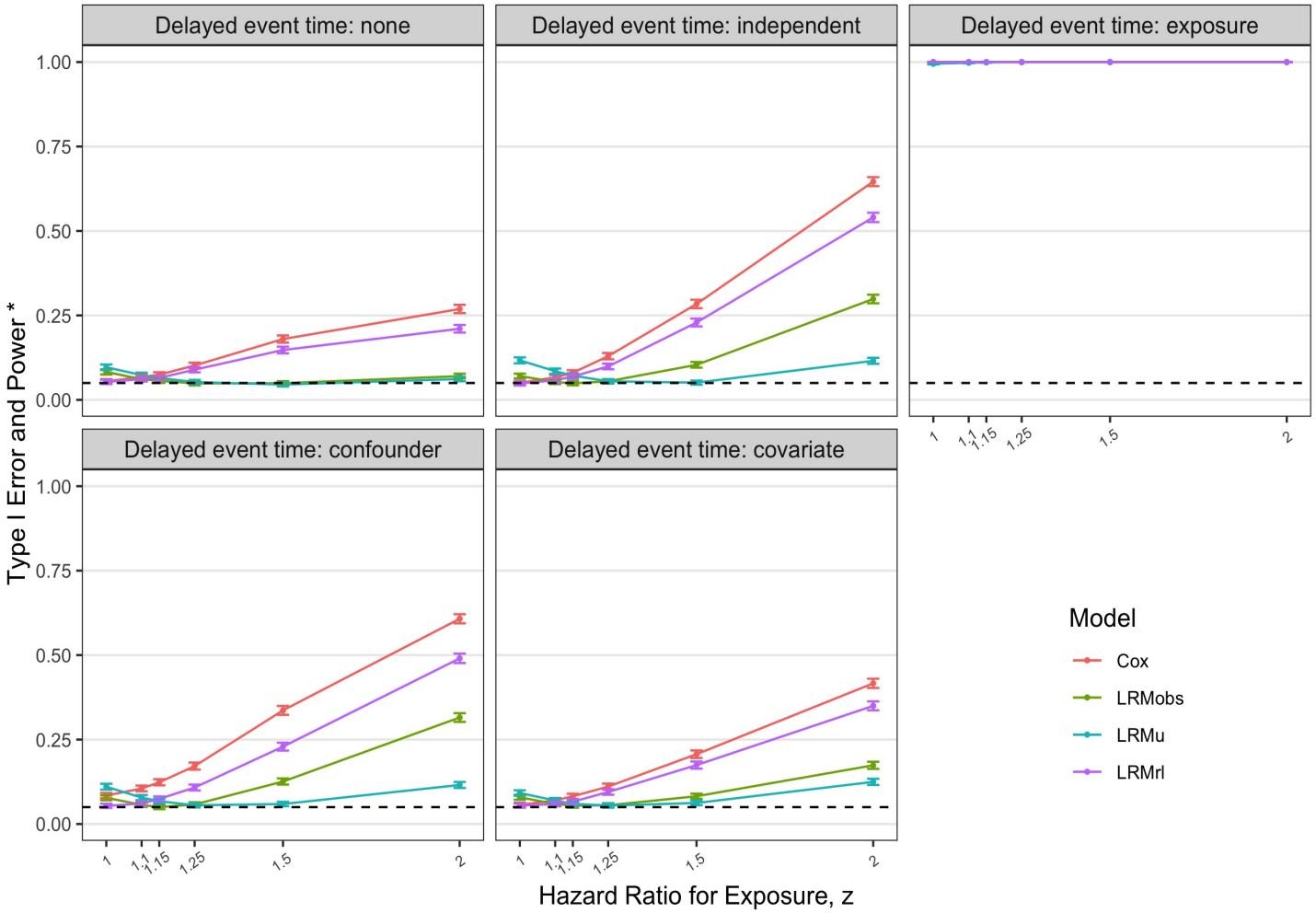

Fig. S4: Results from Simulation 1 (with truncation) when the event time was generated from a Cox model with baseline hazard from an exponential distribution, the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x and z, with left truncation, and the observations with simulated event times before truncation were considered as censored in the analysis (*censor-practice*). The parameters led to a large number of observations with a misclassified event status (detailed in Appendix A).

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

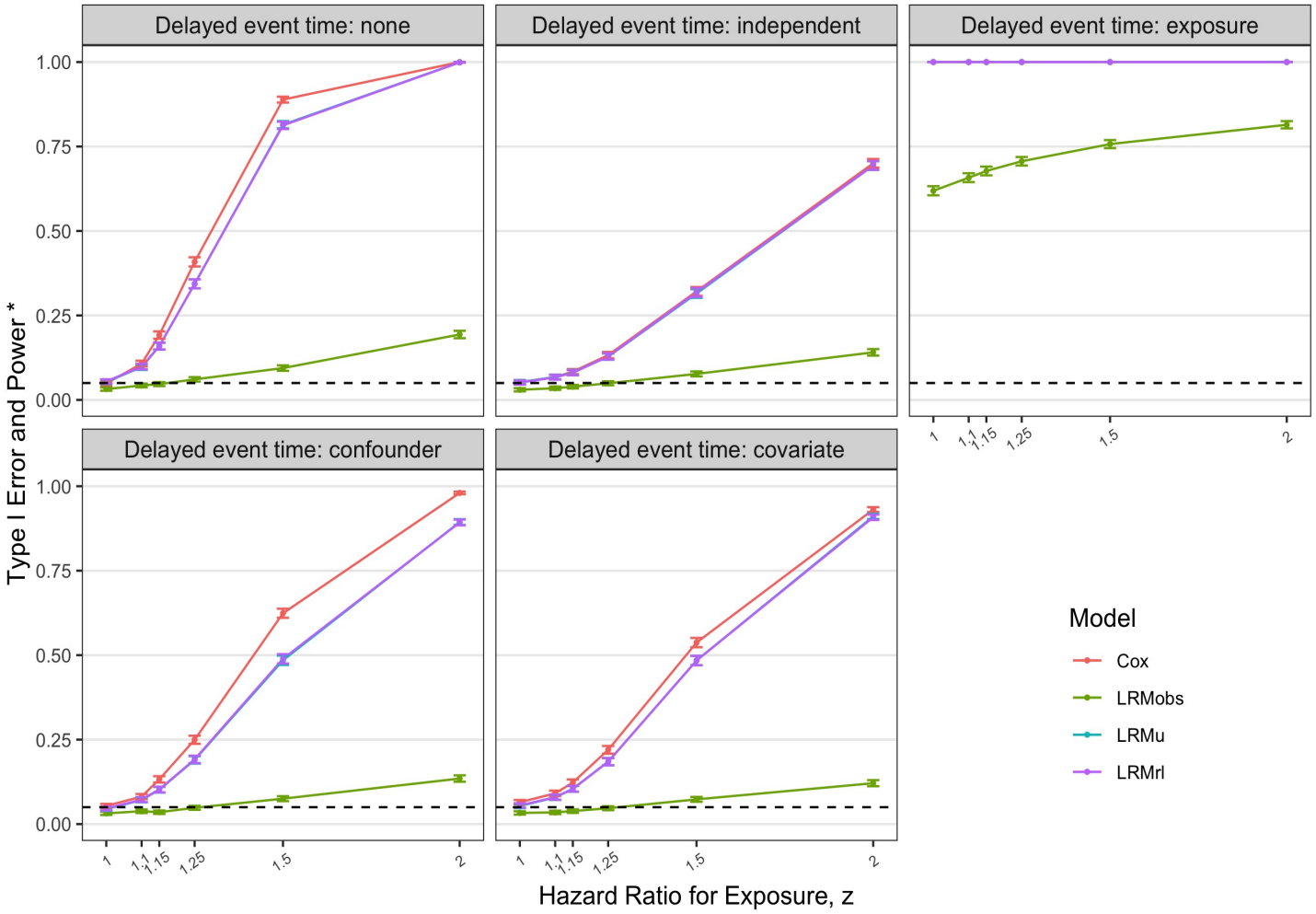

Fig. S5: Results from Simulation 1 (no truncation) when the event time was generated from a Cox model with baseline hazard from an exponential distribution and the censoring time was generated from a uniform distribution. The parameters led to a large number of observations with a misclassified event status (detailed in Appendix A).

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

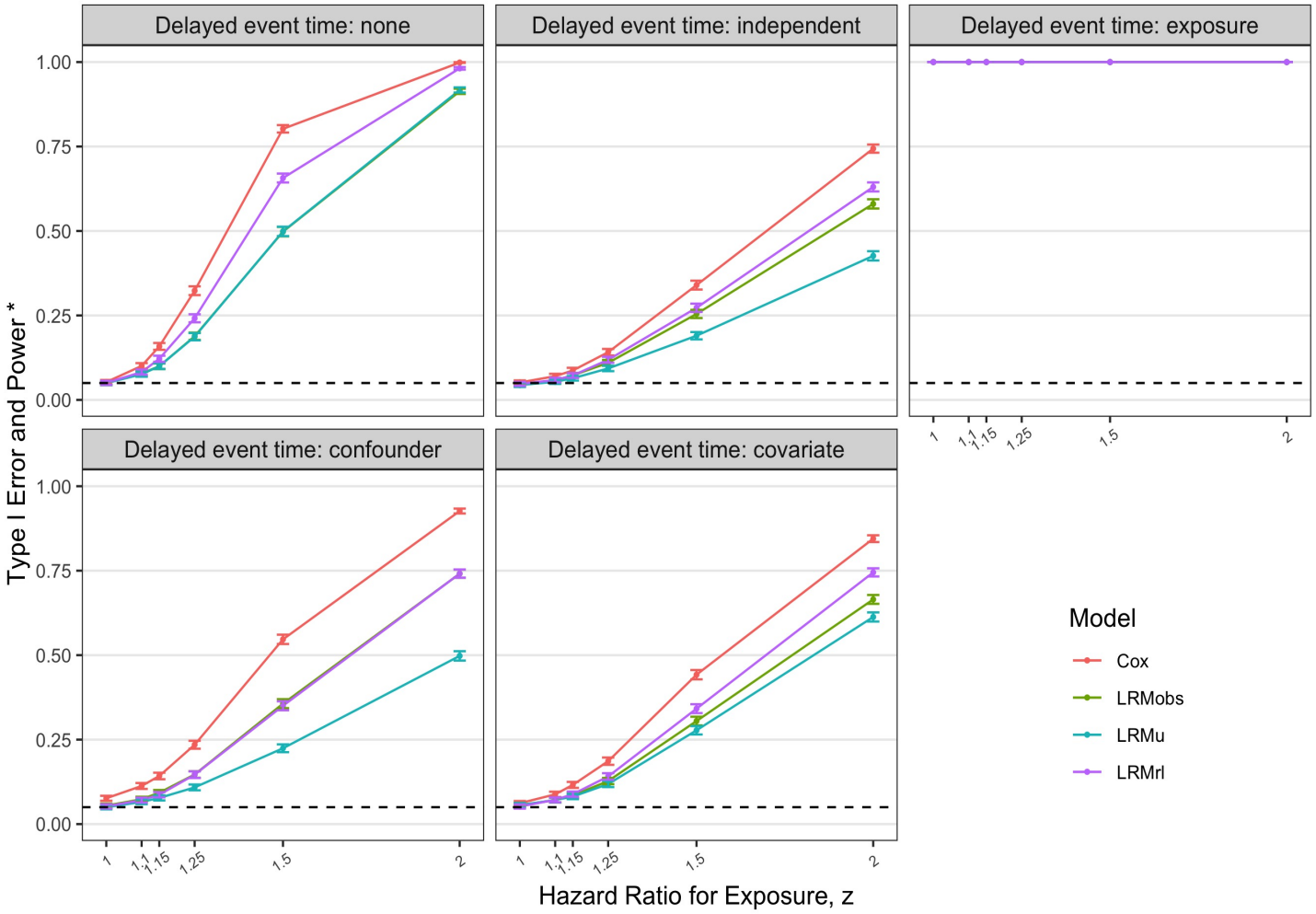

Fig. S6: Results from Simulation 1 (no truncation) when the event time was generated from a Cox model with baseline hazard from an exponential distribution and the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x. The parameters led to a large number of observations with a misclassified event status (detailed in Appendix A).

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

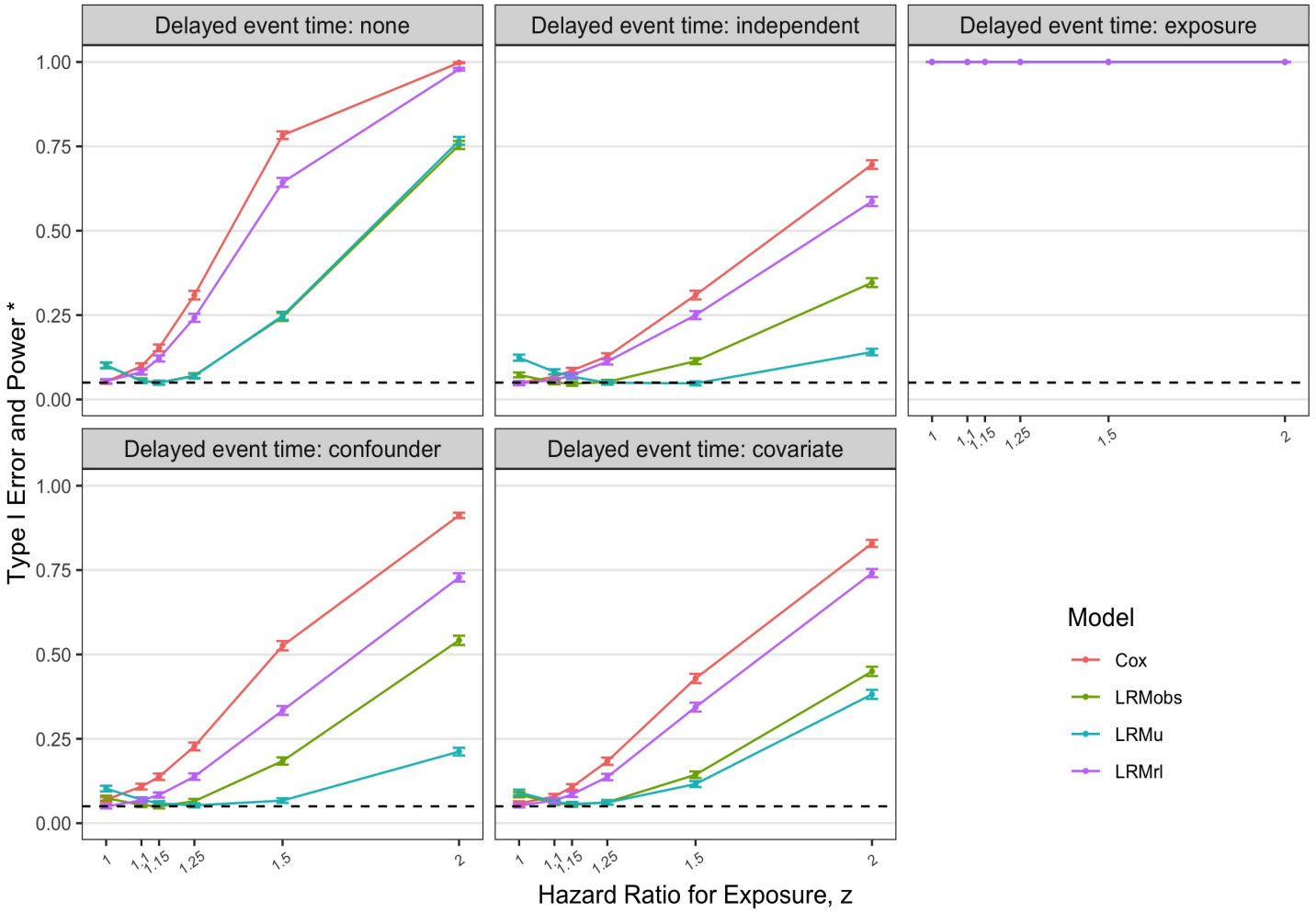

Fig. S7: Results from Simulation 1 (no truncation) when the event time was generated from a Cox model with baseline hazard from an exponential distribution and the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x and z. The parameters led to a large number of observations with a misclassified event status (detailed in Appendix A).

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

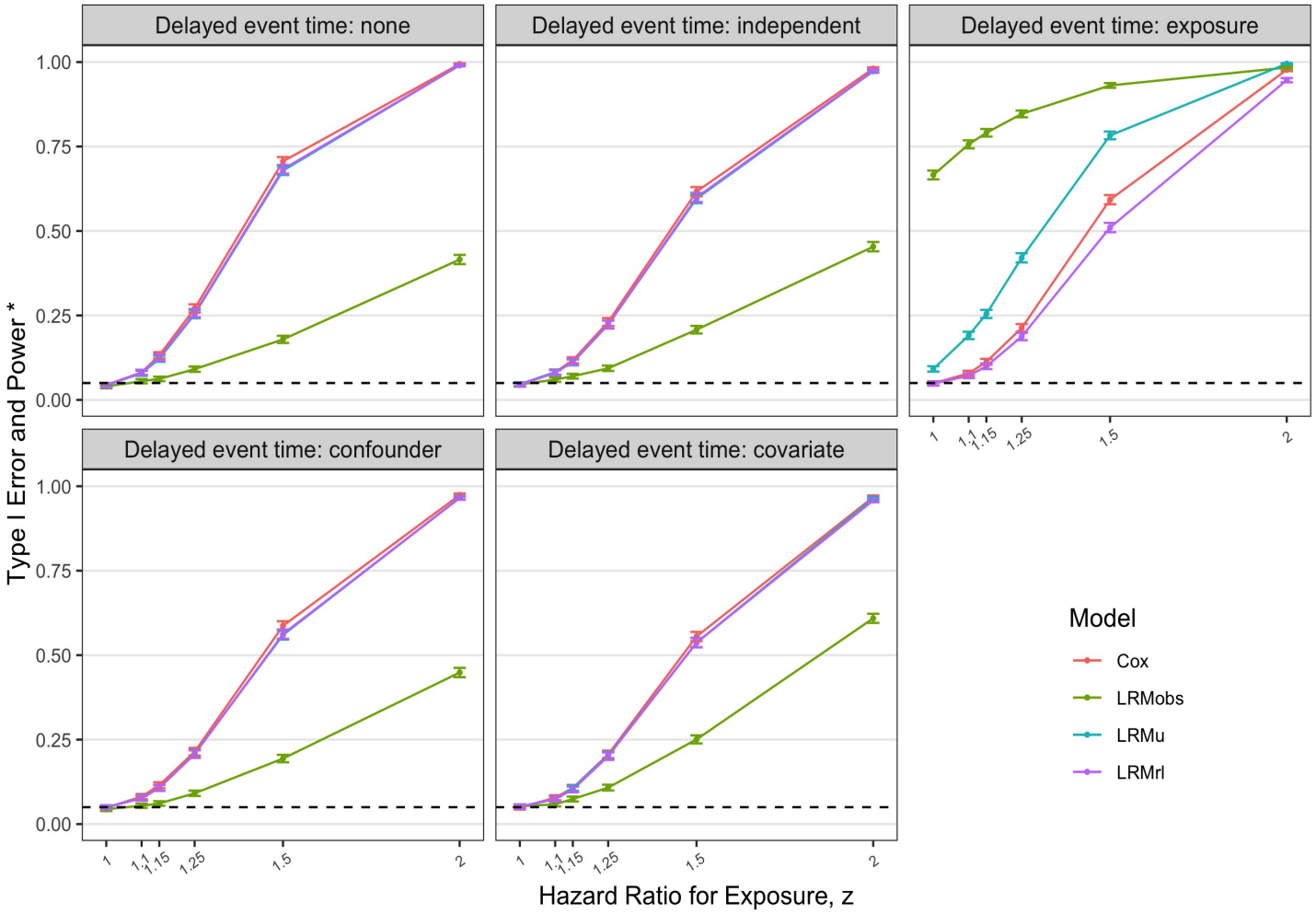

Fig. S8: Results from Simulation 2 when the event time was generated from a Cox model with baseline hazard from an exponential distribution and the censoring time was generated from a uniform distribution. The parameters are the same as those in Figure 1 of main paper.
* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

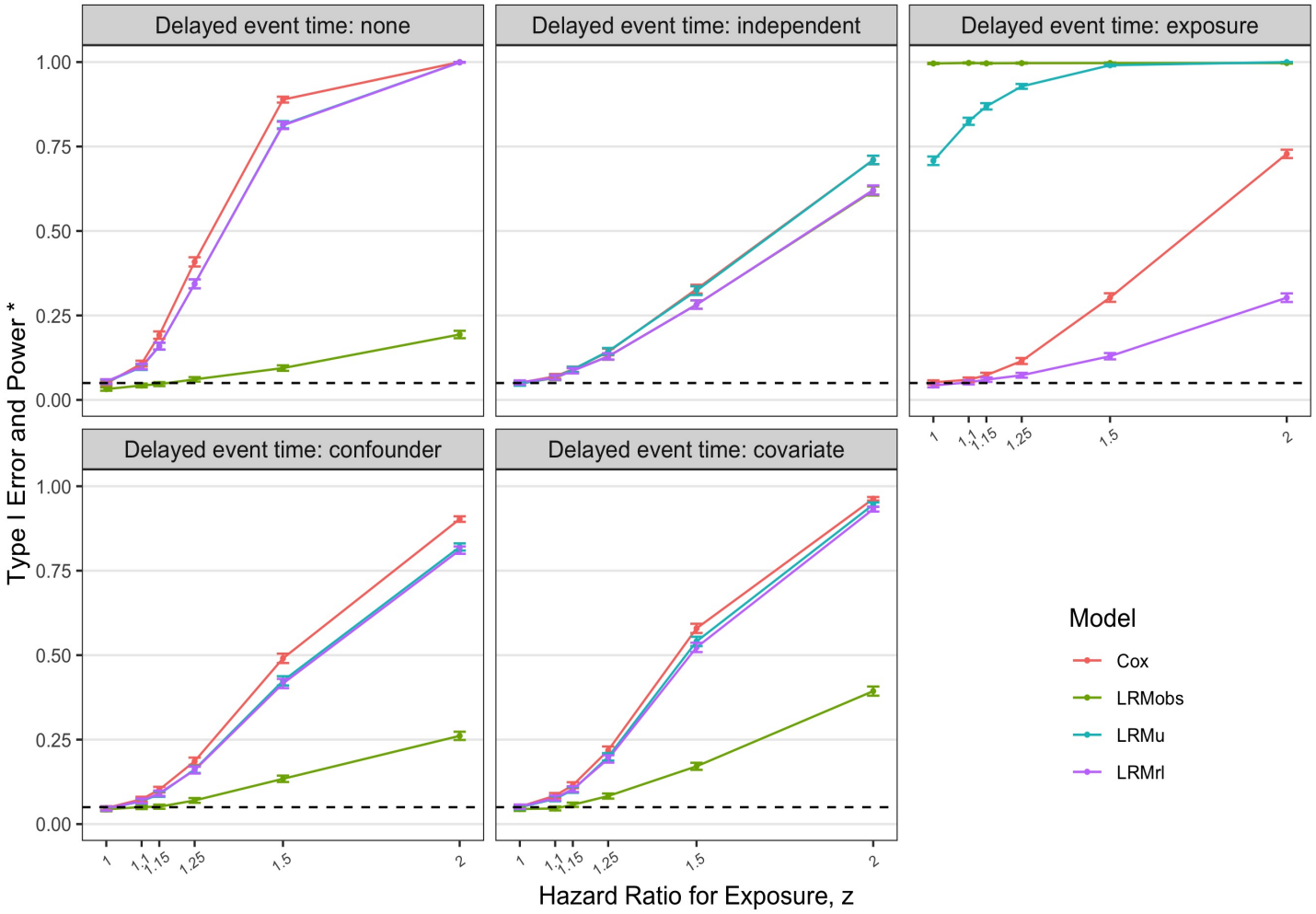

Fig. S9: Results from Simulation 2 when the event time was generated from a Cox model with baseline hazard from an exponential distribution and the censoring time was generated from a uniform distribution. The parameters are the same as those in Figure 2 of main paper.
* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

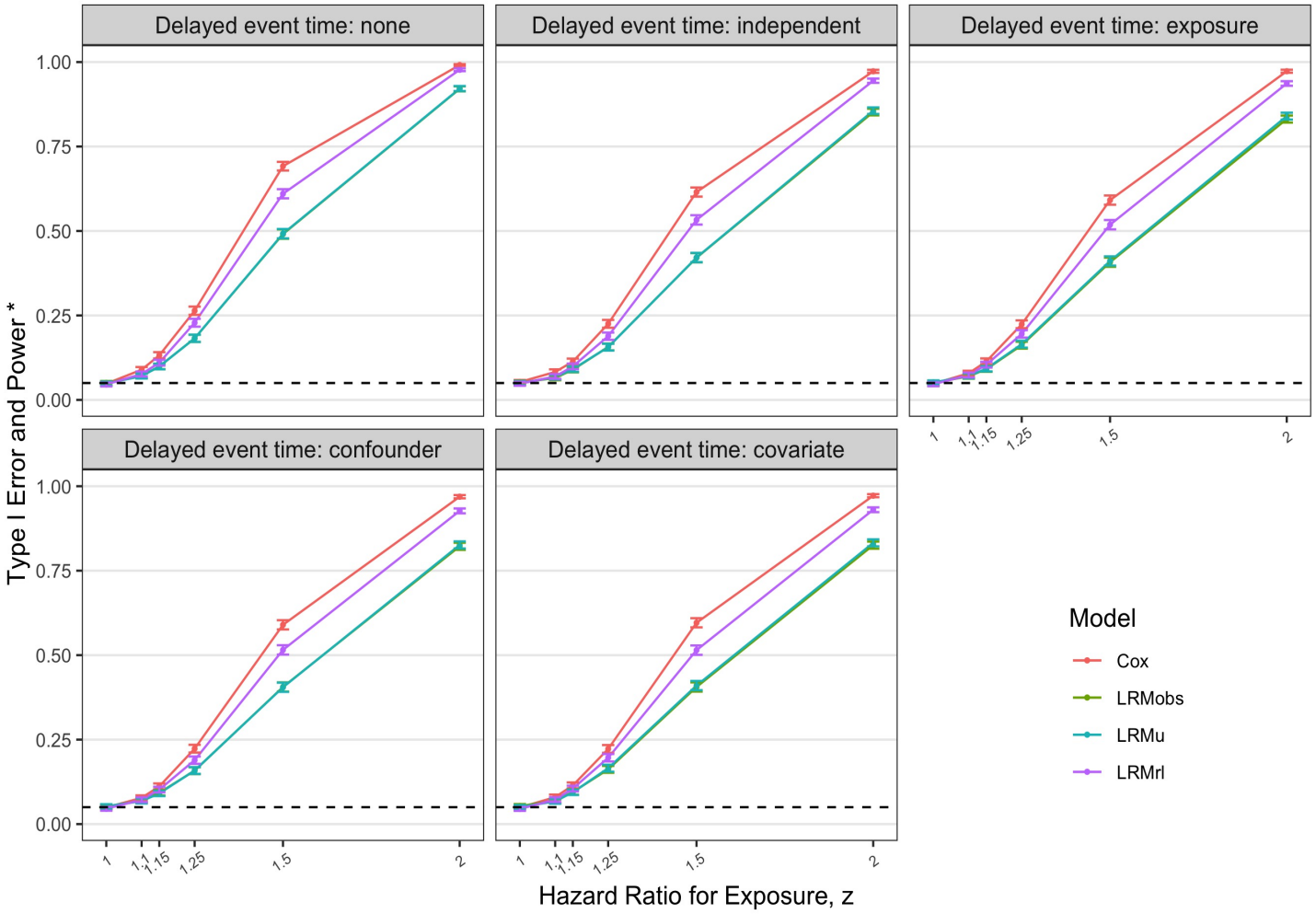

Fig. S10: Results from Simulation 2 when the event time was generated from a Cox model with baseline hazard from an exponential distribution and the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x. The parameters are the same as those in Figure 1 of main paper.

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

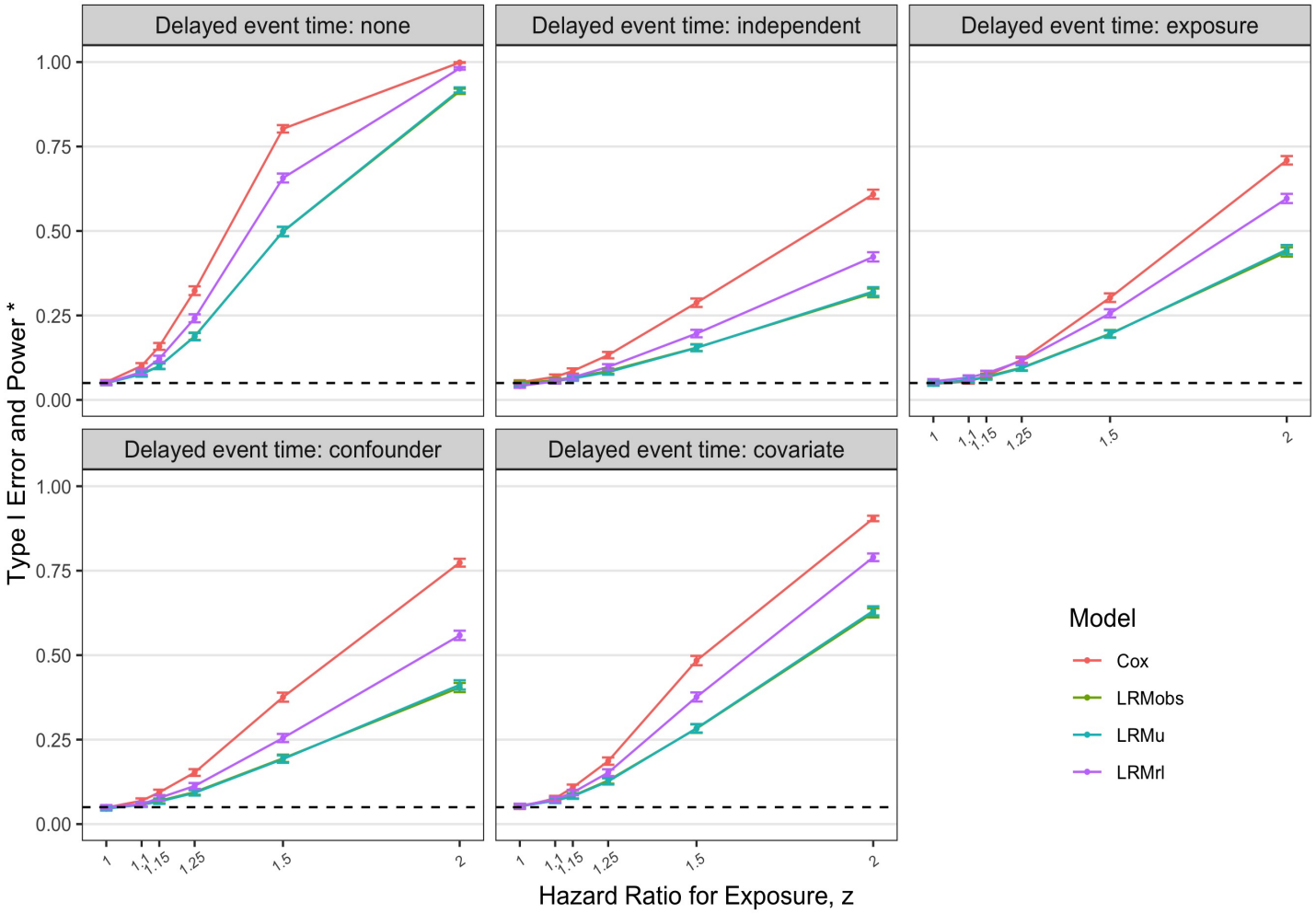

Fig. S11: Results from Simulation 2 when the event time was generated from a Cox model with baseline hazard from an exponential distribution and the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x. The parameters are the same as those in Figure 2 of main paper.

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

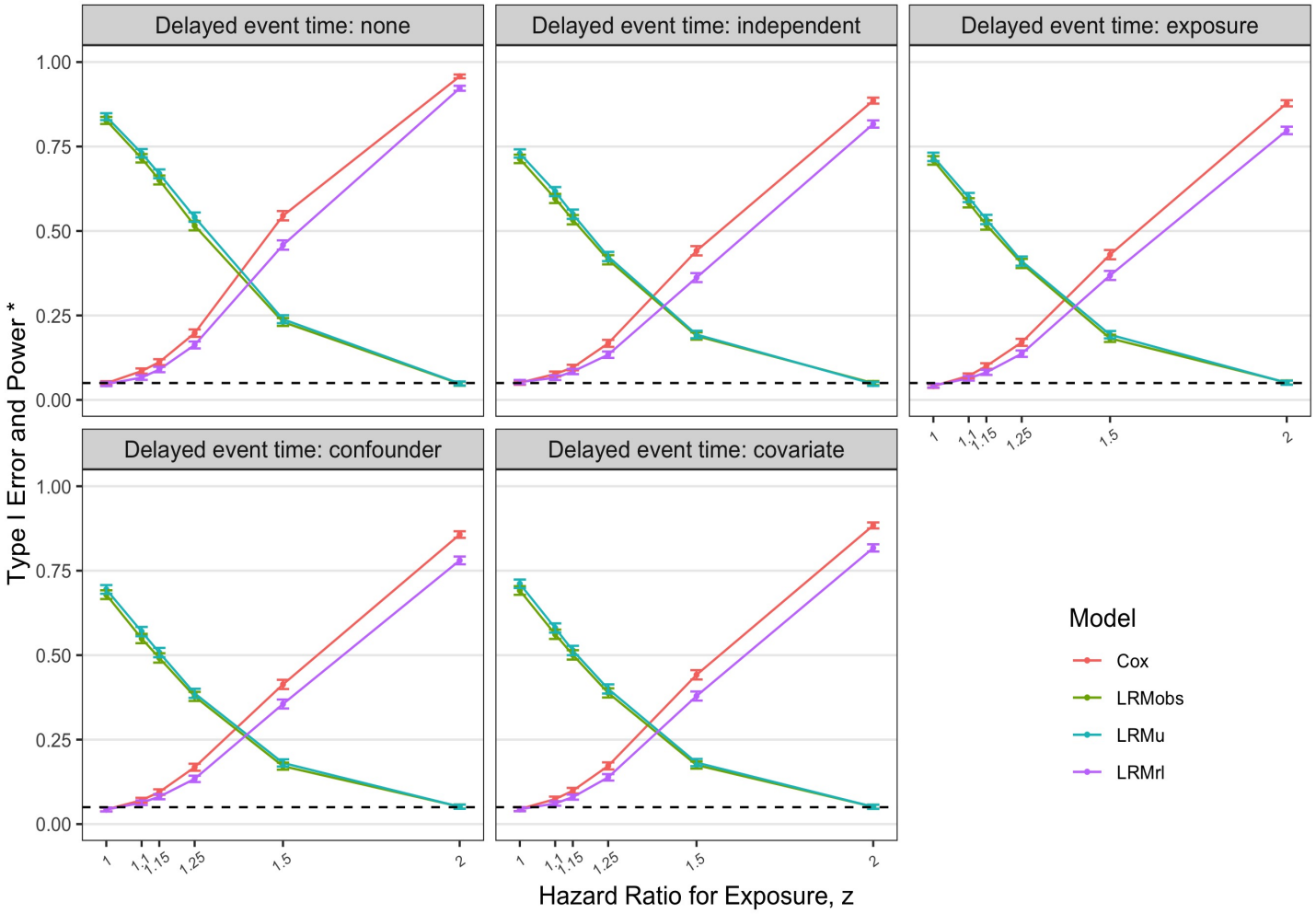

Fig. S12: Results from Simulation 2 when the event time was generated from a Cox model with baseline hazard from an exponential distribution and the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x and z. The parameters are the same as those in Figure 1 of main paper.

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

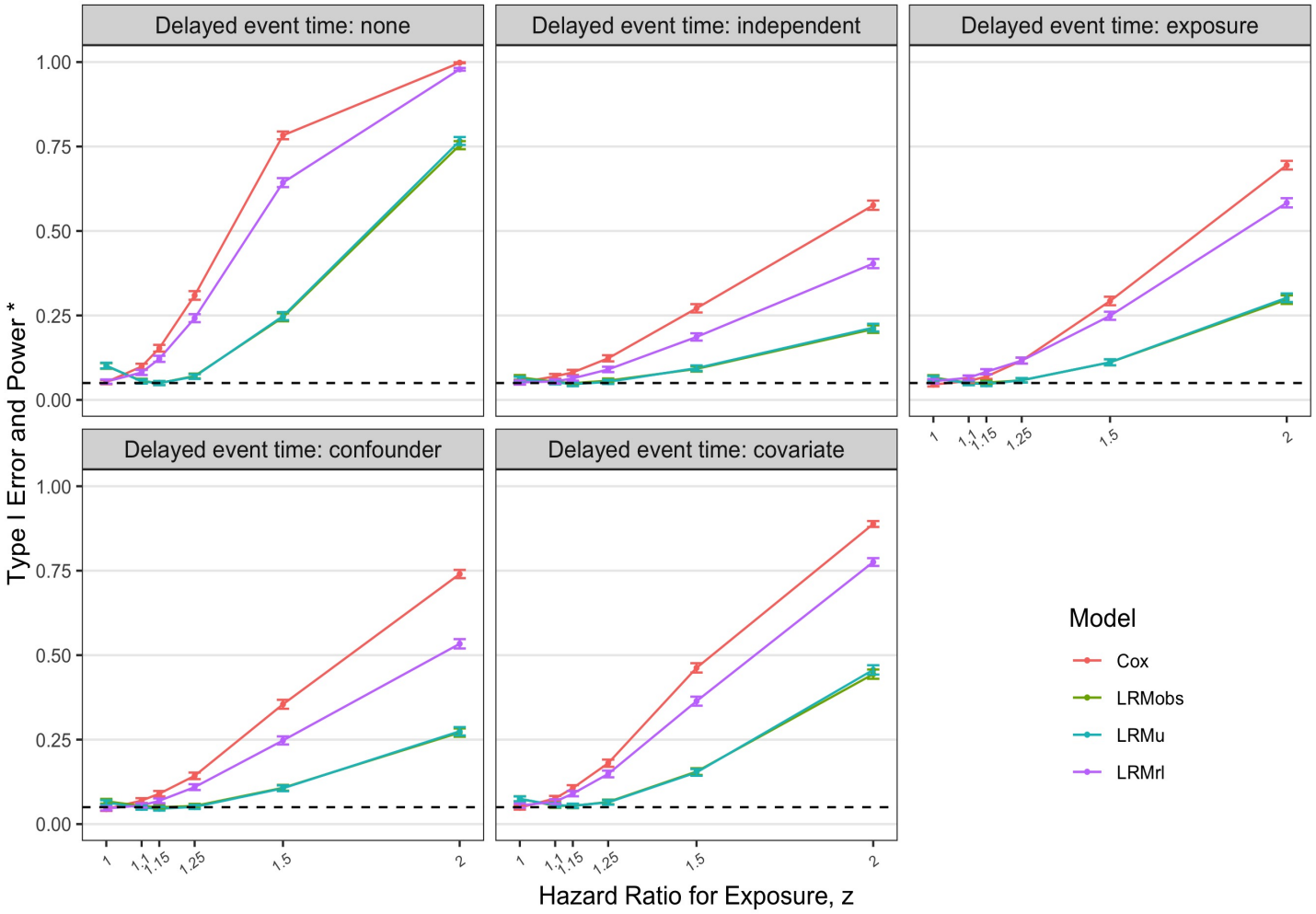

Fig. S13: Results from Simulation 2 when the event time was generated from a Cox model with baseline hazard from an exponential distribution and the censoring time was generated from a Cox model with baseline hazard from an exponential distribution that depended on x and z. The parameters are the same as those in Figure 2 of main paper.

* Type I error evaluated at log(1). Power evaluated at log(1.1), log(1.15), log(1.25), log(1.5), log(2).

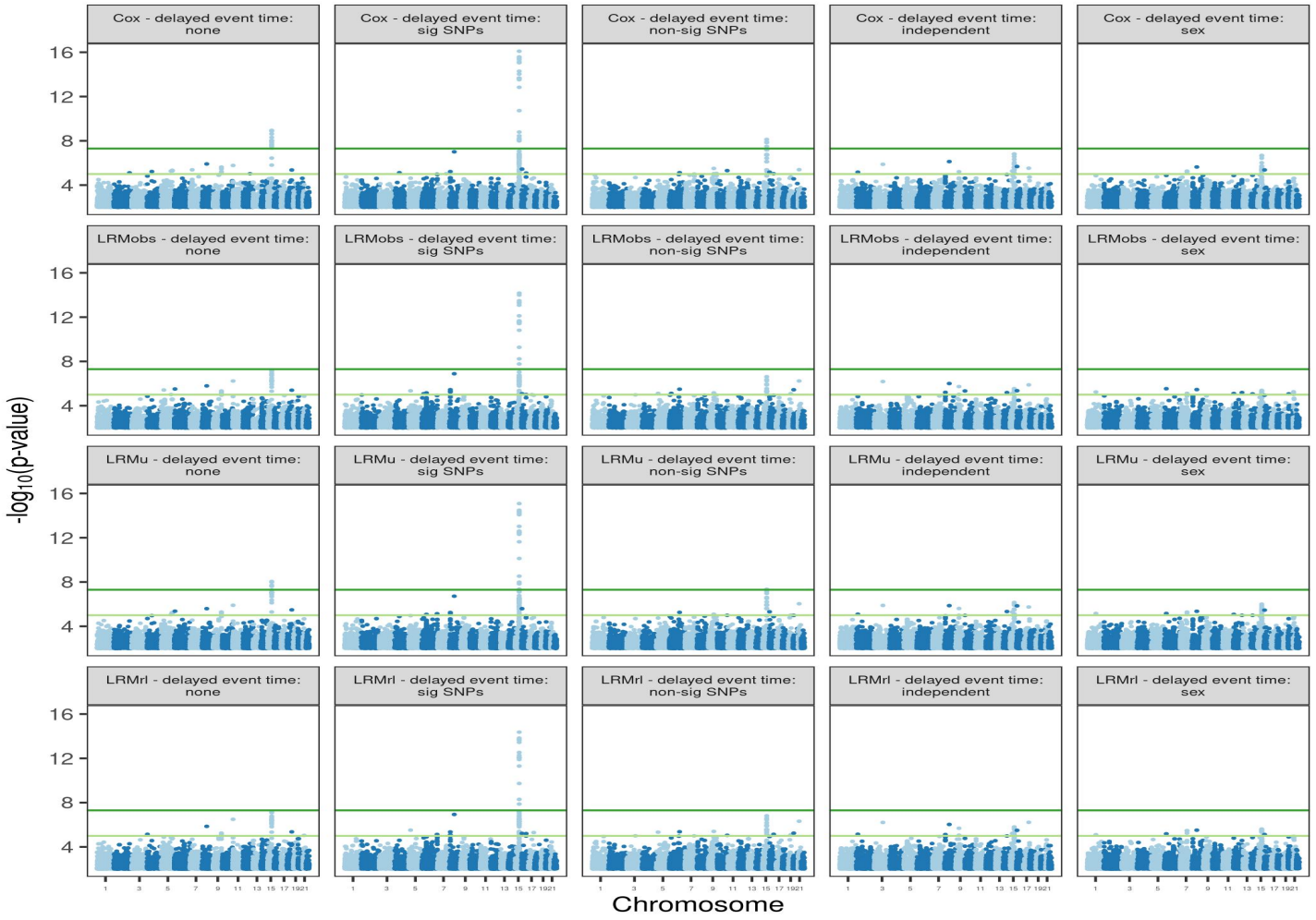

Fig. S14: Manhattan plots of GWAS results for cancer of bronchus; lung (phecode 165.1) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

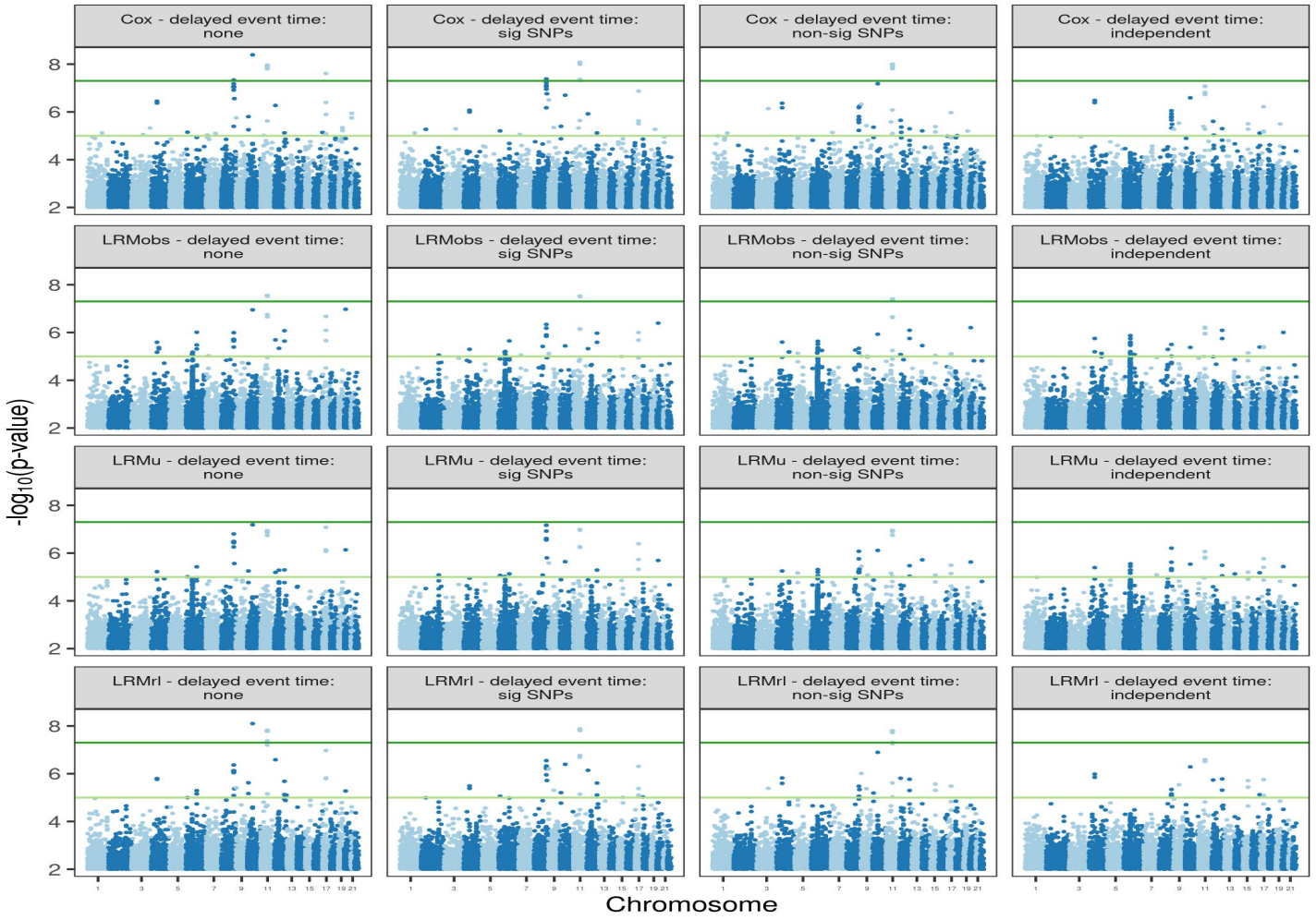

Fig. S15: Manhattan plots of GWAS results for cancer of prostate (phecode 185) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

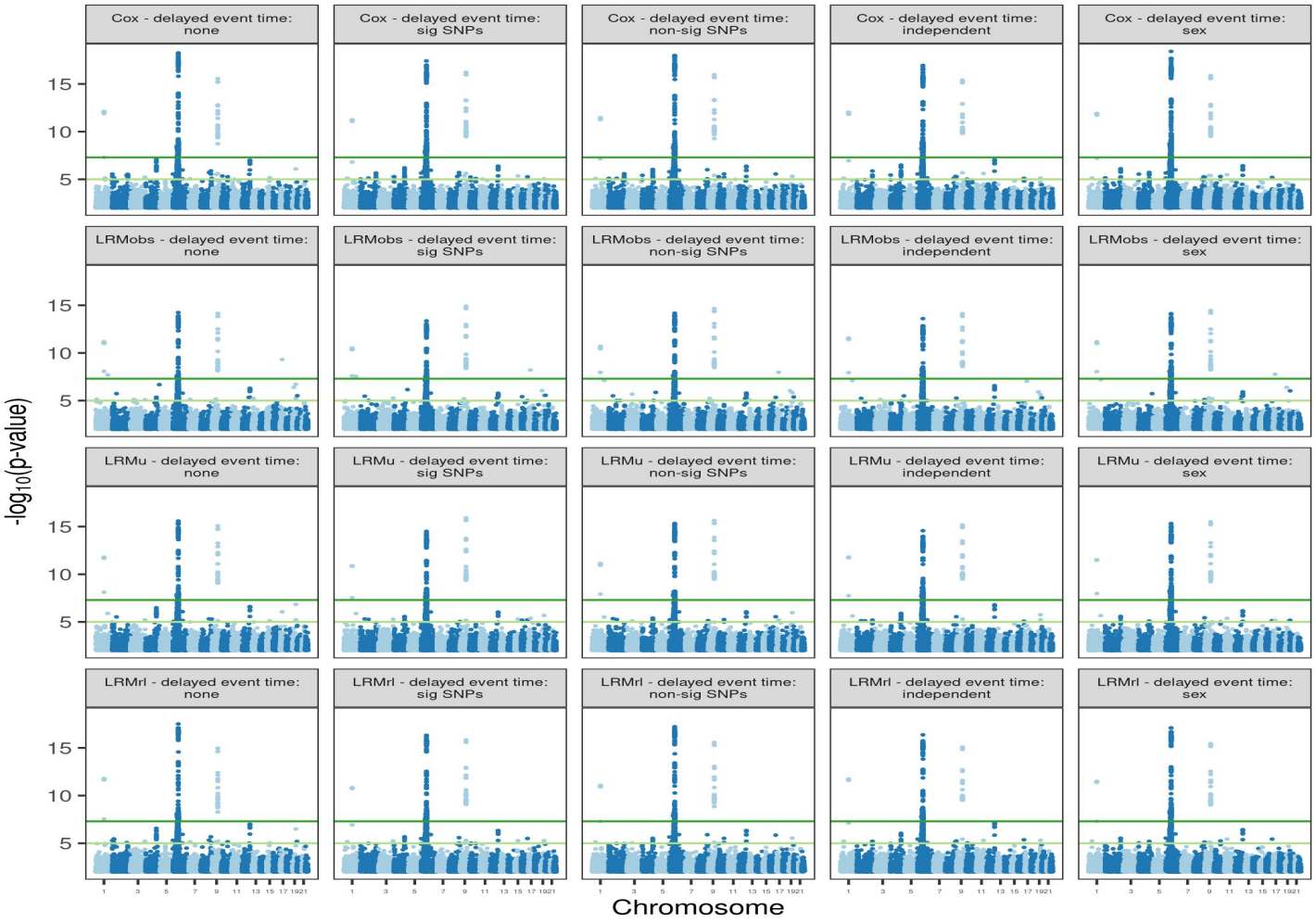

Fig. S16: Manhattan plots of GWAS results for hypothyroidism (phecode 244) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

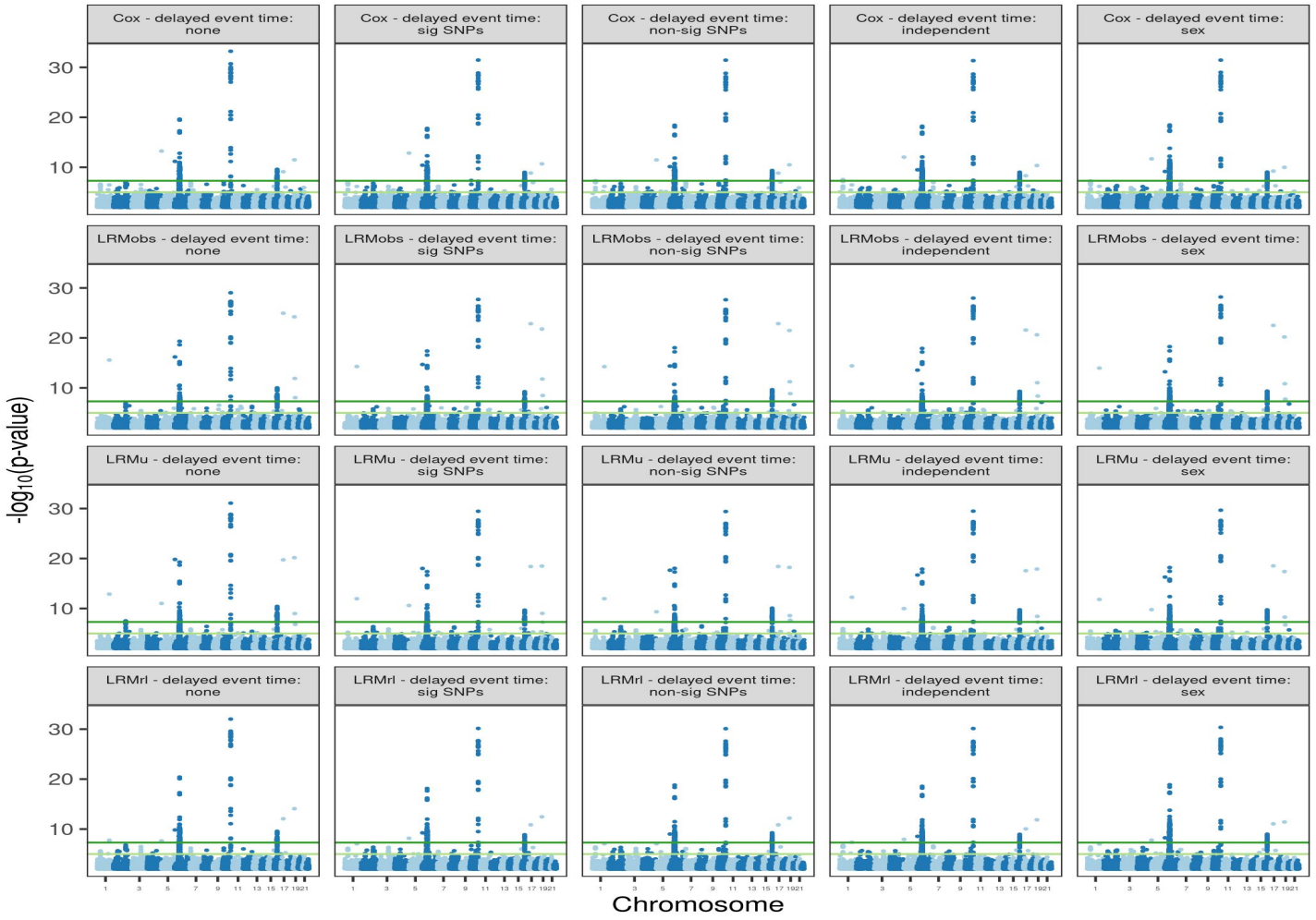

Fig. S17: Manhattan plots of GWAS results for type 2 diabetes (phecode 250.2) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

Fig. S18: Manhattan plots of GWAS results for vitamin D deficiency (phecode 261.4) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

Fig. S19: Manhattan plots of GWAS results for hypercholesterolemia (phecode 272.11) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

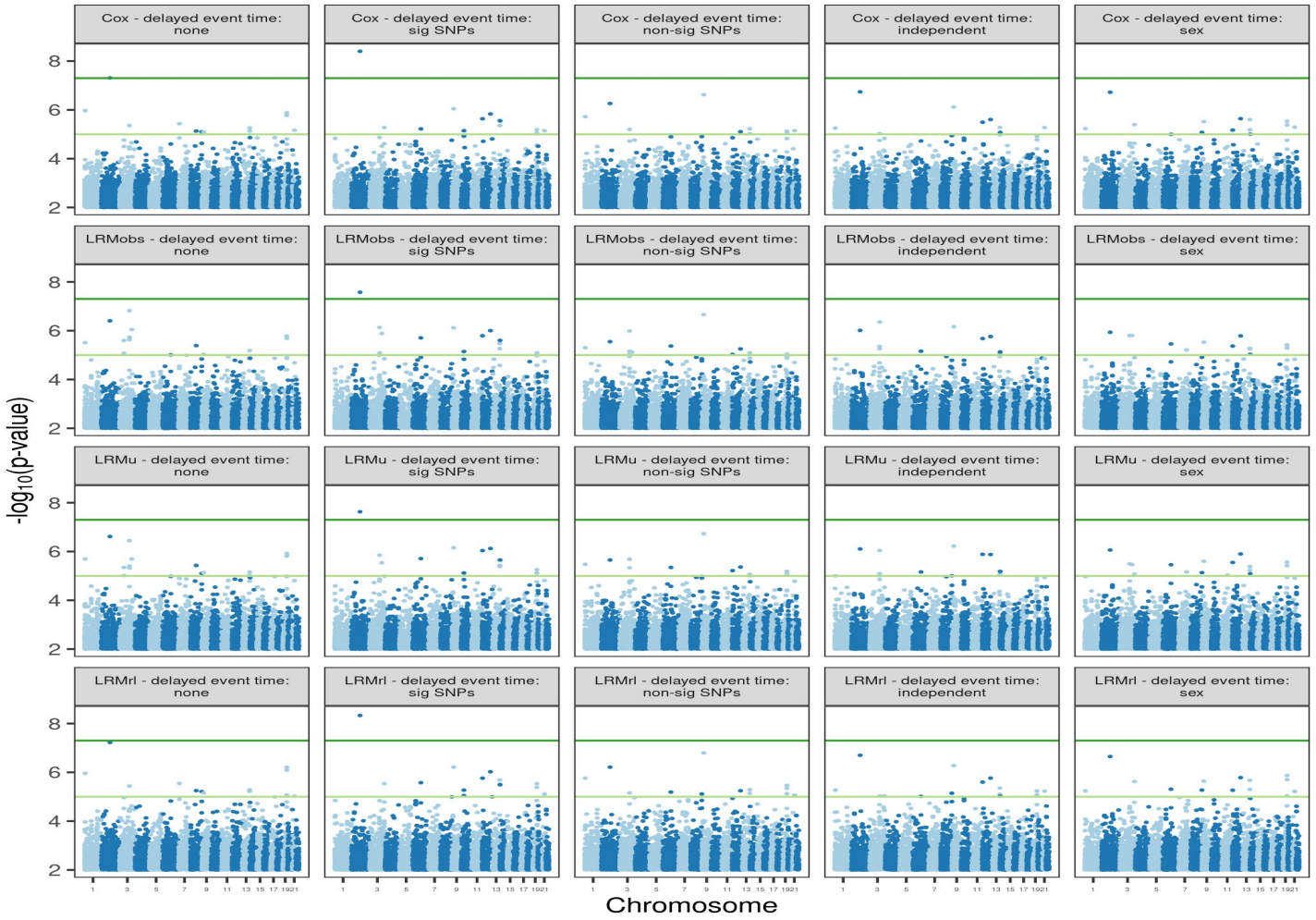

Fig. S20: Manhattan plots of GWAS results for insomnia (phecode 327.4) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

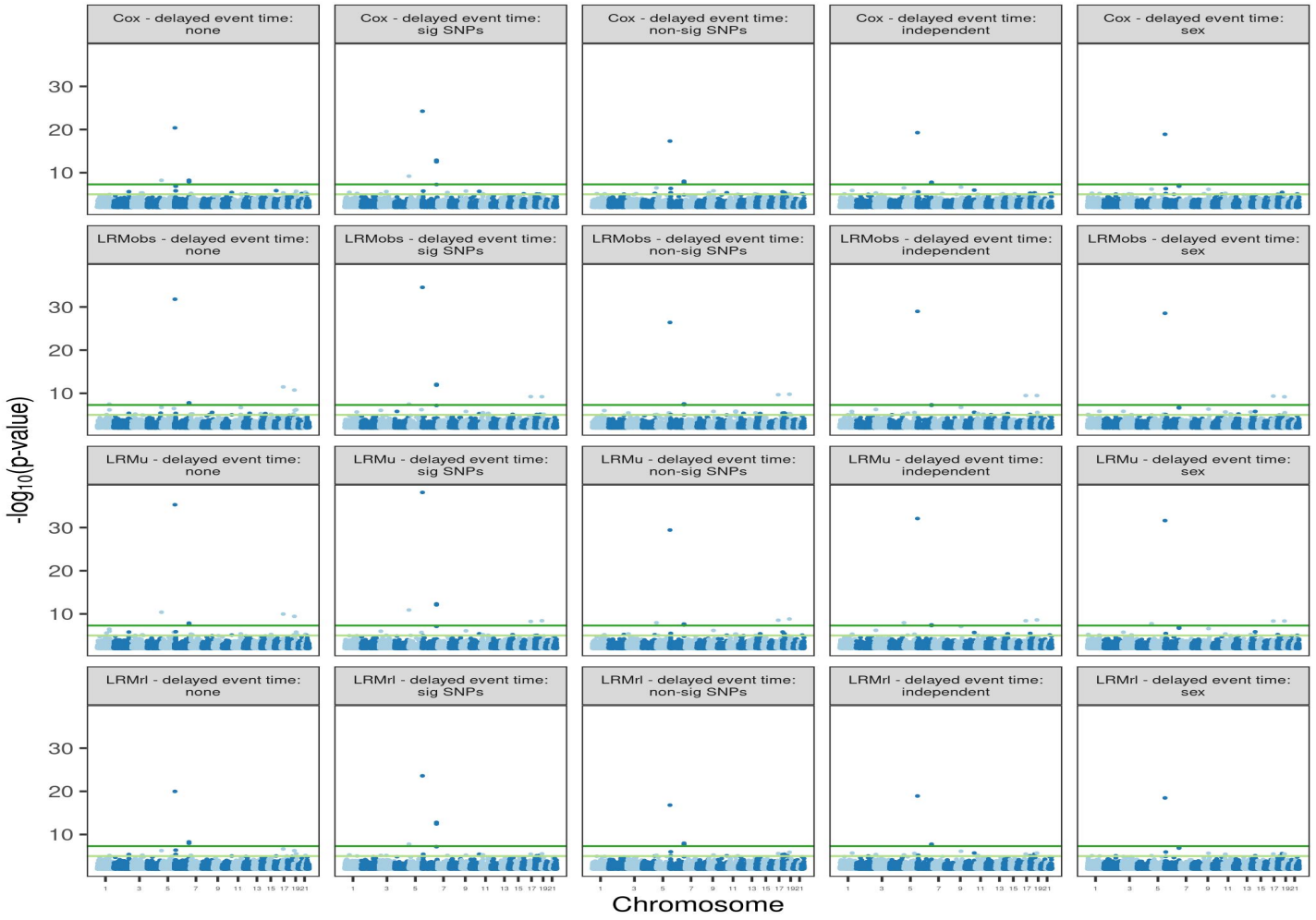

Fig. S21: Manhattan plots of GWAS results for myocardial infarction (phecode 411.2) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

Fig. S22: Manhattan plots of GWAS results for coronary atherosclerosis (phecode 411.4) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

Fig. S23: Manhattan plots of GWAS results for atrial fibrillation (phecode 427.21) for each model and delayed event time combination. The dark green line corresponds to $P \leq 5 \times 10^{-8}$ and the light green line corresponds to $P \leq 1 \times 10^{-5}$.

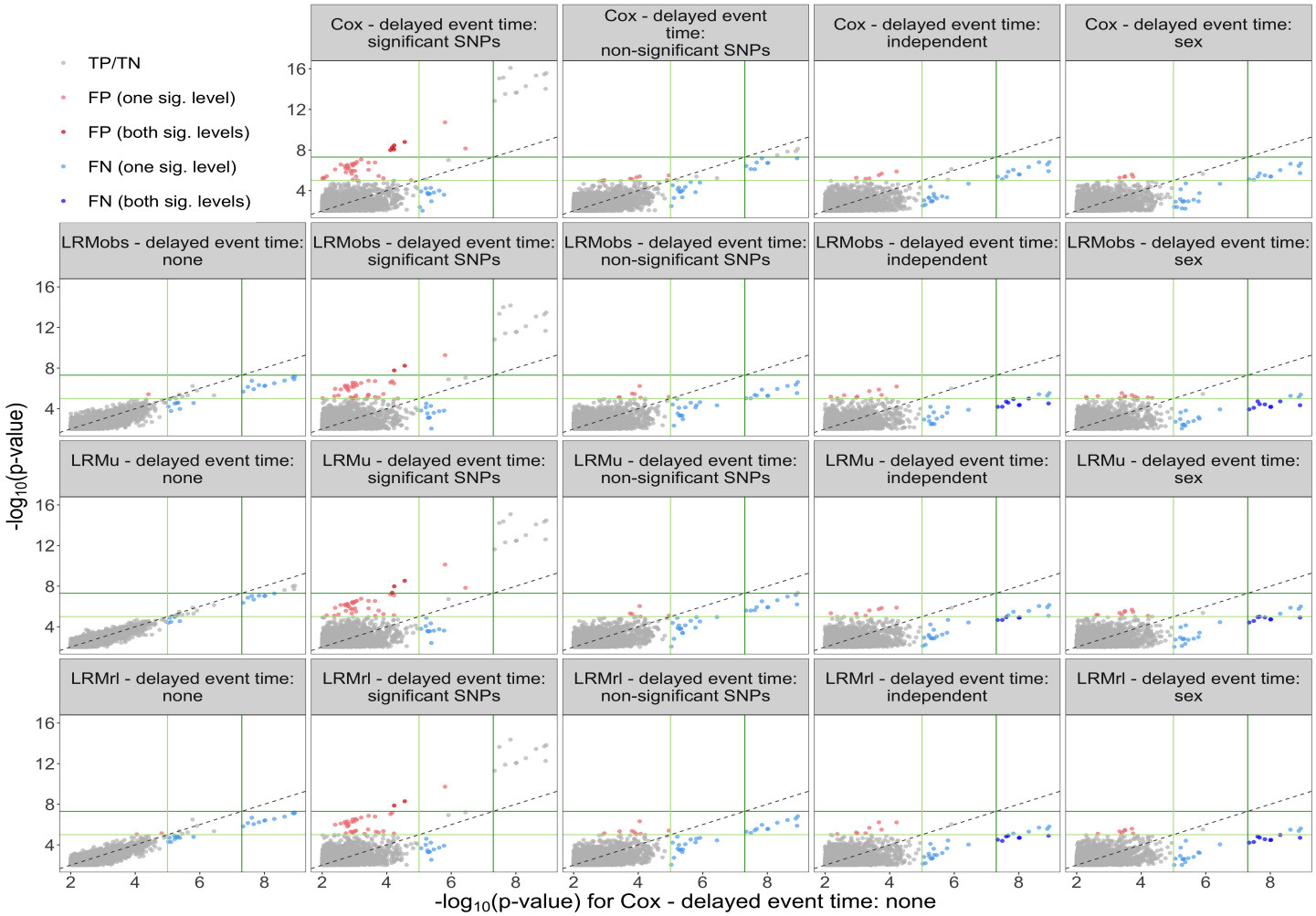

Fig. S24: False positive and false negative SNPs for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for cancer of bronchus; lung (phecode 165.1). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.

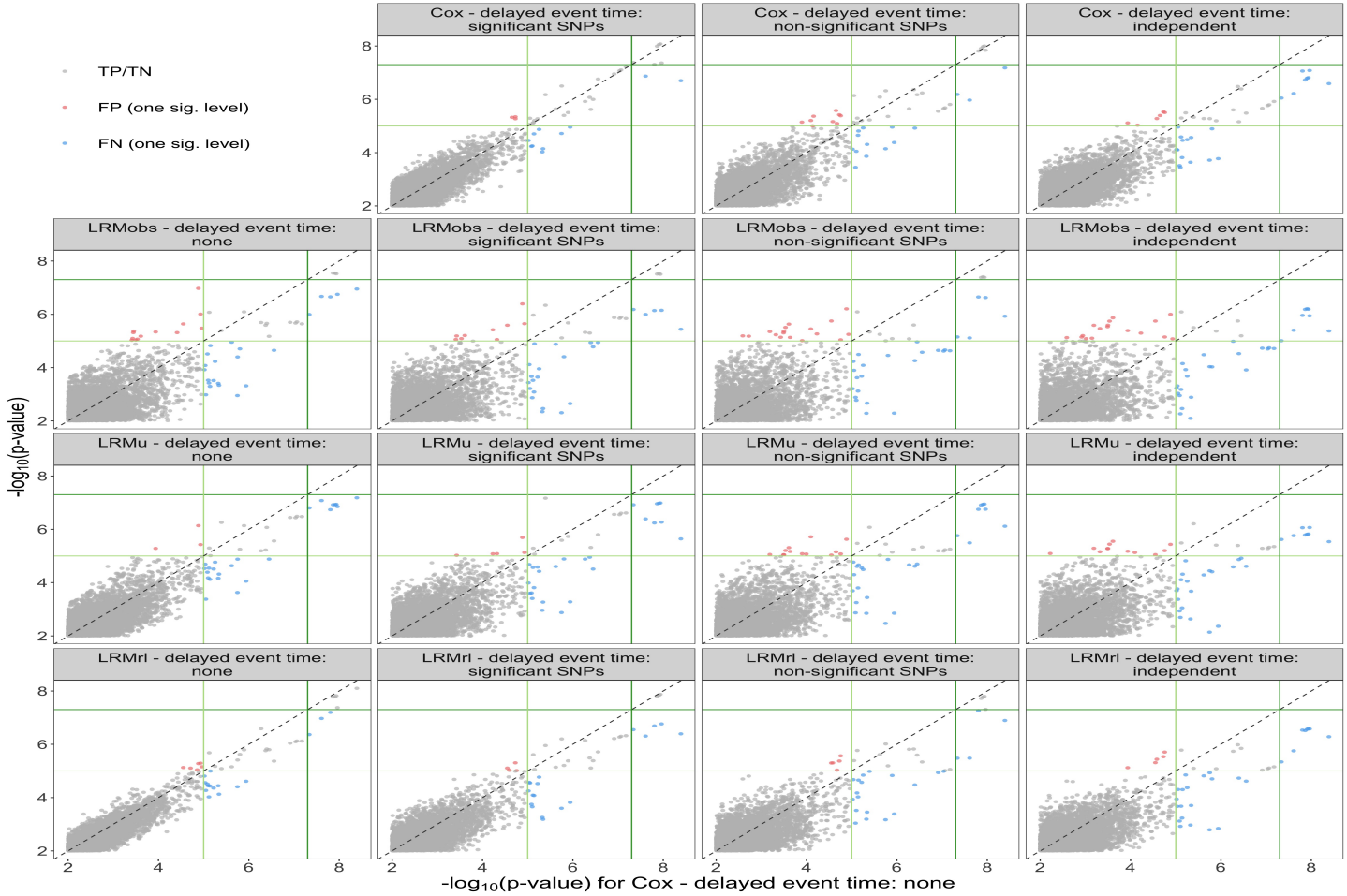

Fig. S25: False positive and false negative SNPs for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for cancer of prostate (phecode 185). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.

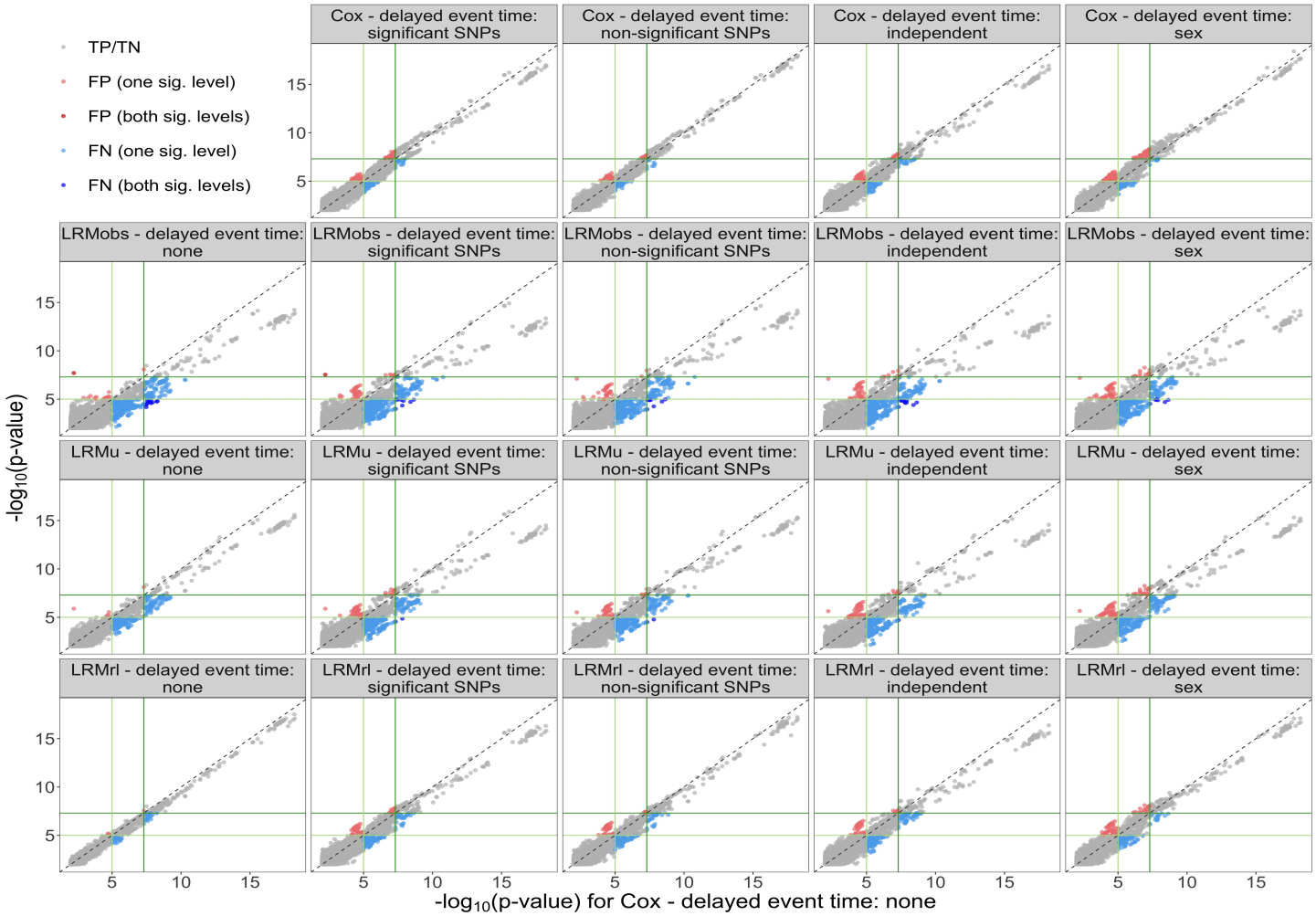

Fig. S26: False positive and false negative SNPs for each model and delayed event time combination, using Model 1 (*Cox*) with no delayed event time as the gold standard, for hypothyroidism (phecode 244). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.

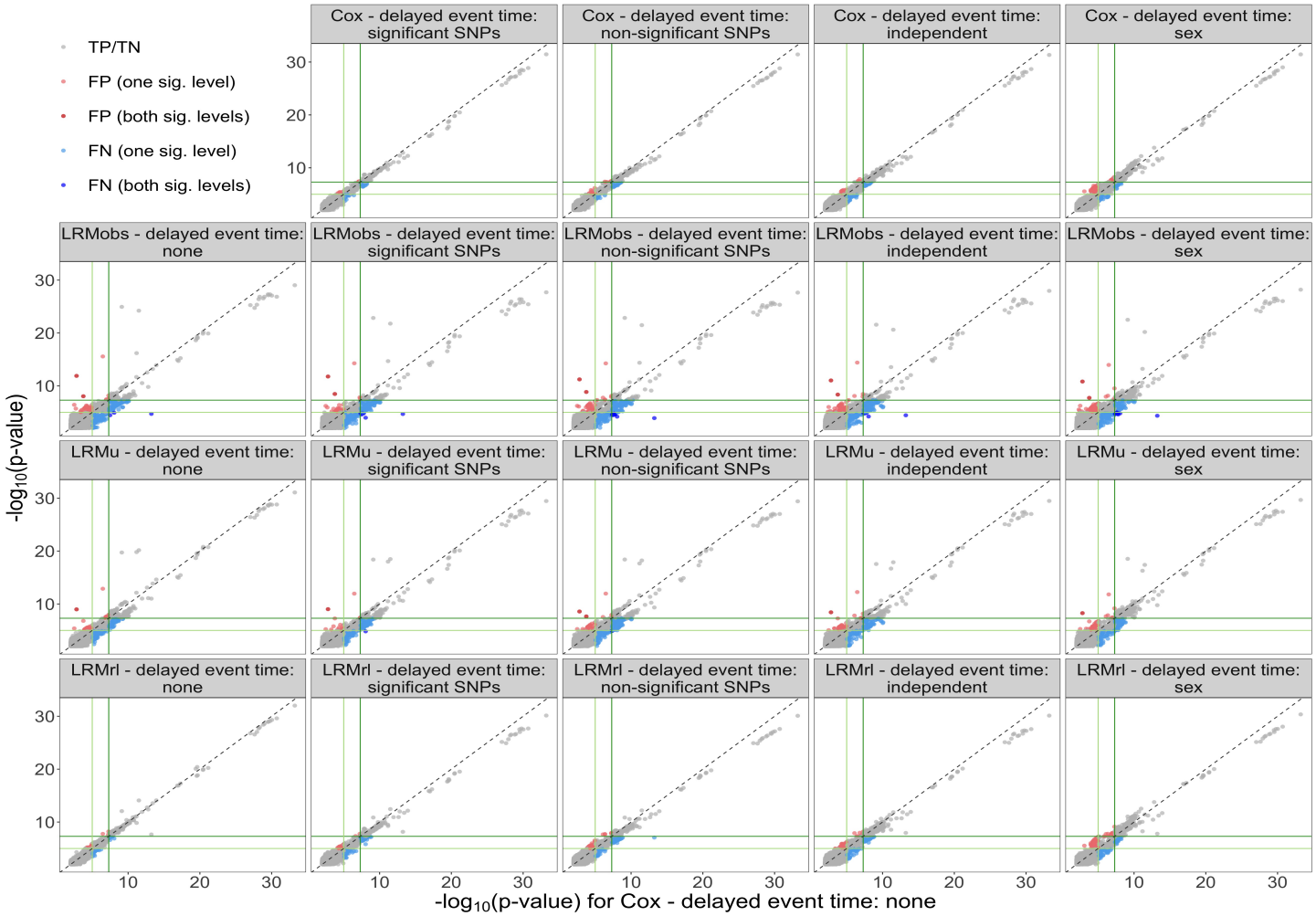

Fig. S27: False positive and false negative SNPs for each model and delayed event time combination, using Model 1 (*Cox*) with no delayed event time as the gold standard, for type 2 diabetes (phecode 250.2). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.

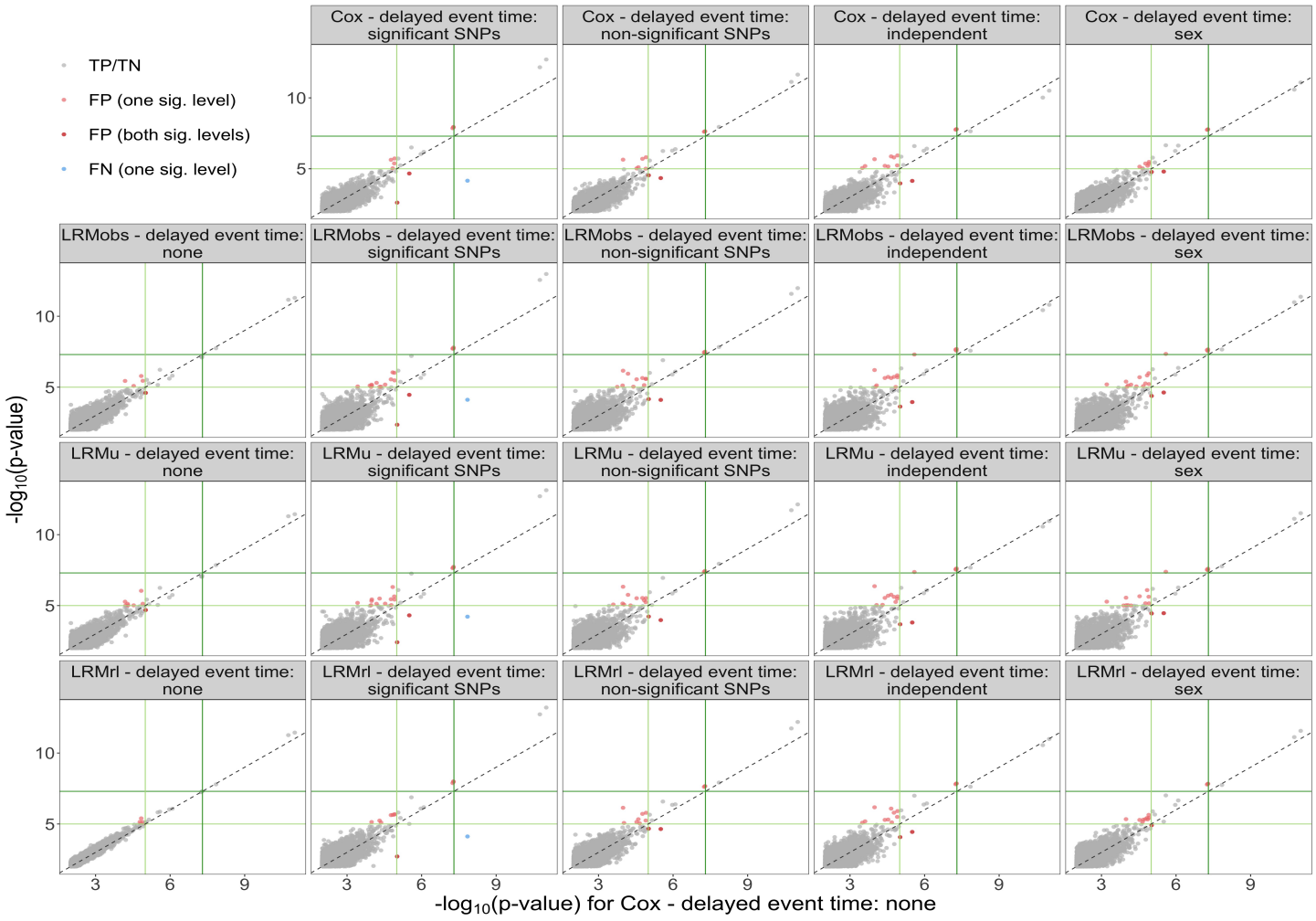

Fig. S28: False negative SNPs for each model and delayed event time combination, using Model 1 (*Cox*) with no delayed event time as the gold standard, for vitamin D deficiency (phecode 261.4). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.

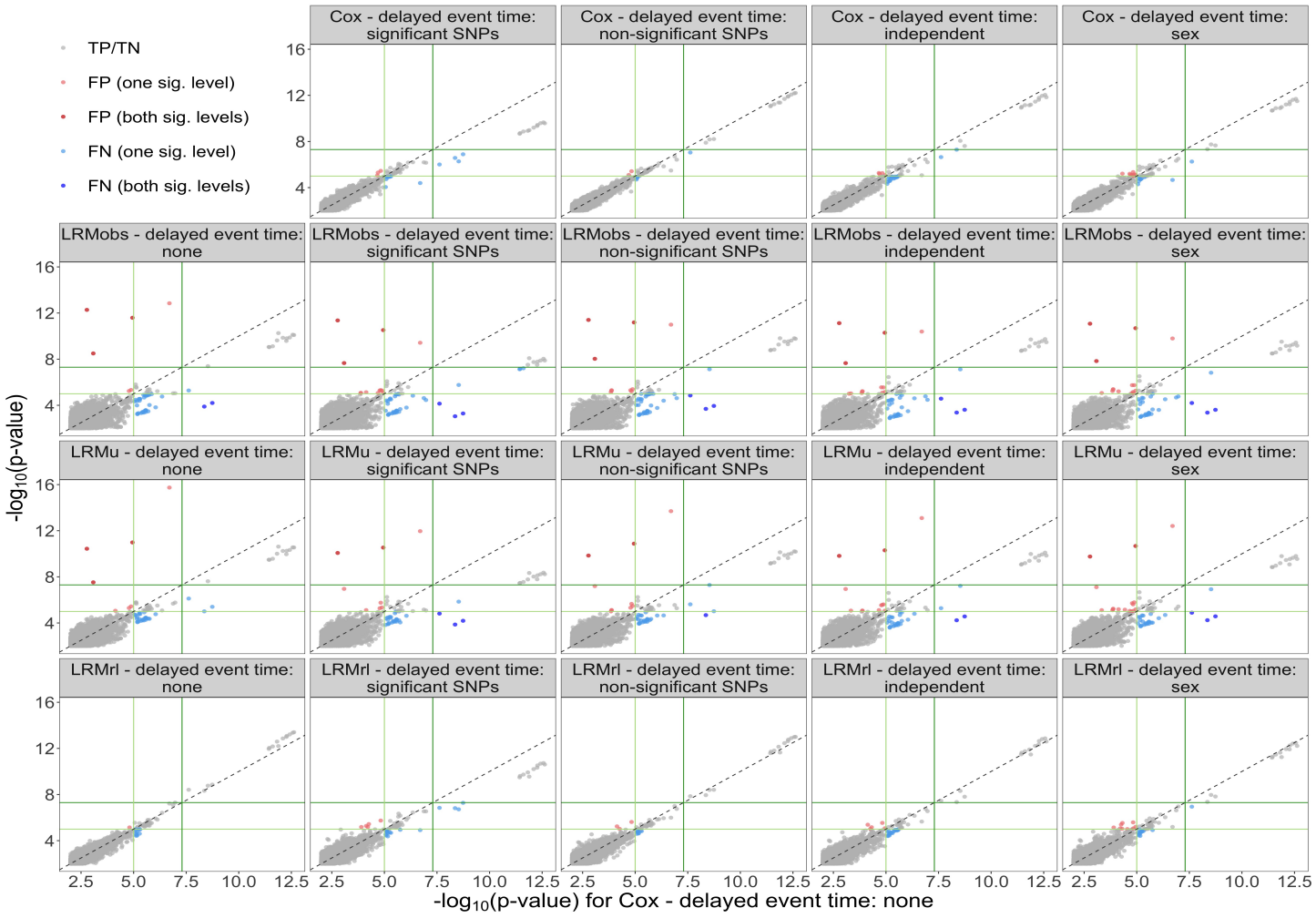

Fig. S29: False negative SNPs for each model and delayed event time combination, using Model 1 (*Cox*) with no delayed event time as the gold standard, for hypercholesterolemia (phecode 272.11). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.

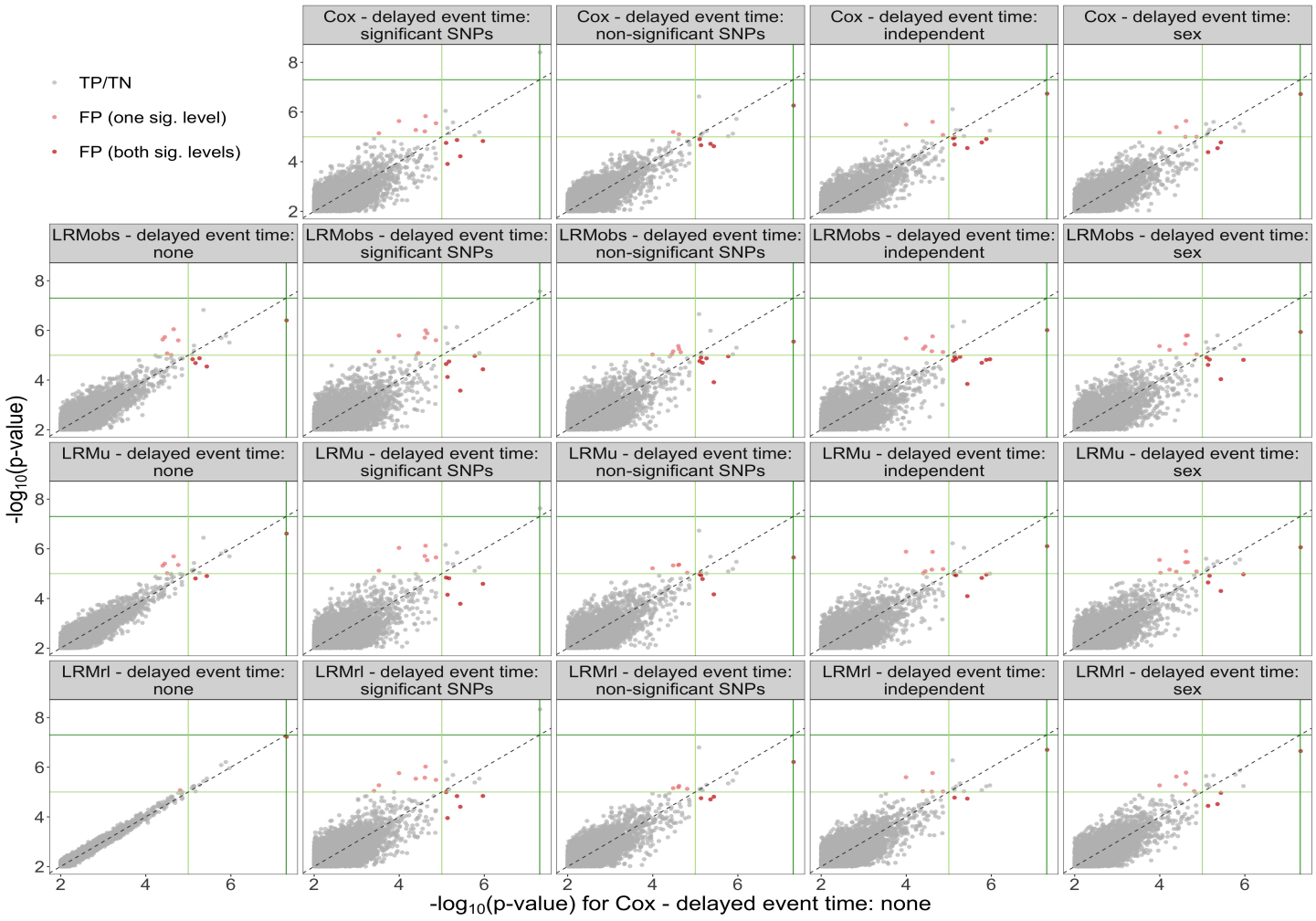

Fig. S30: False positive and false negative SNPs for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for insomnia (phecode 327.4). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.

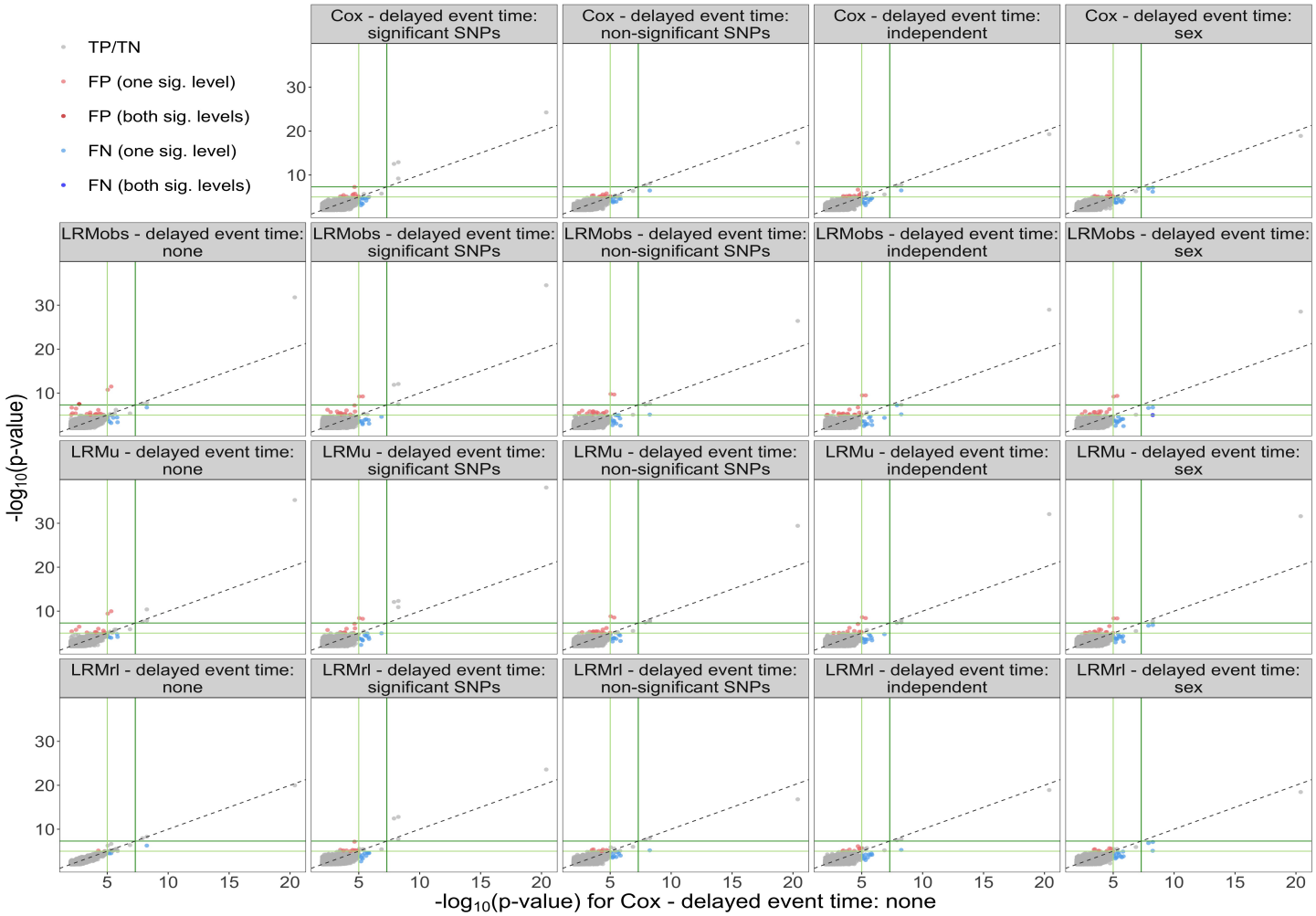

Fig. S31: False positive and false negative SNPs for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for myocardial infarction (phecode 411.2). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.

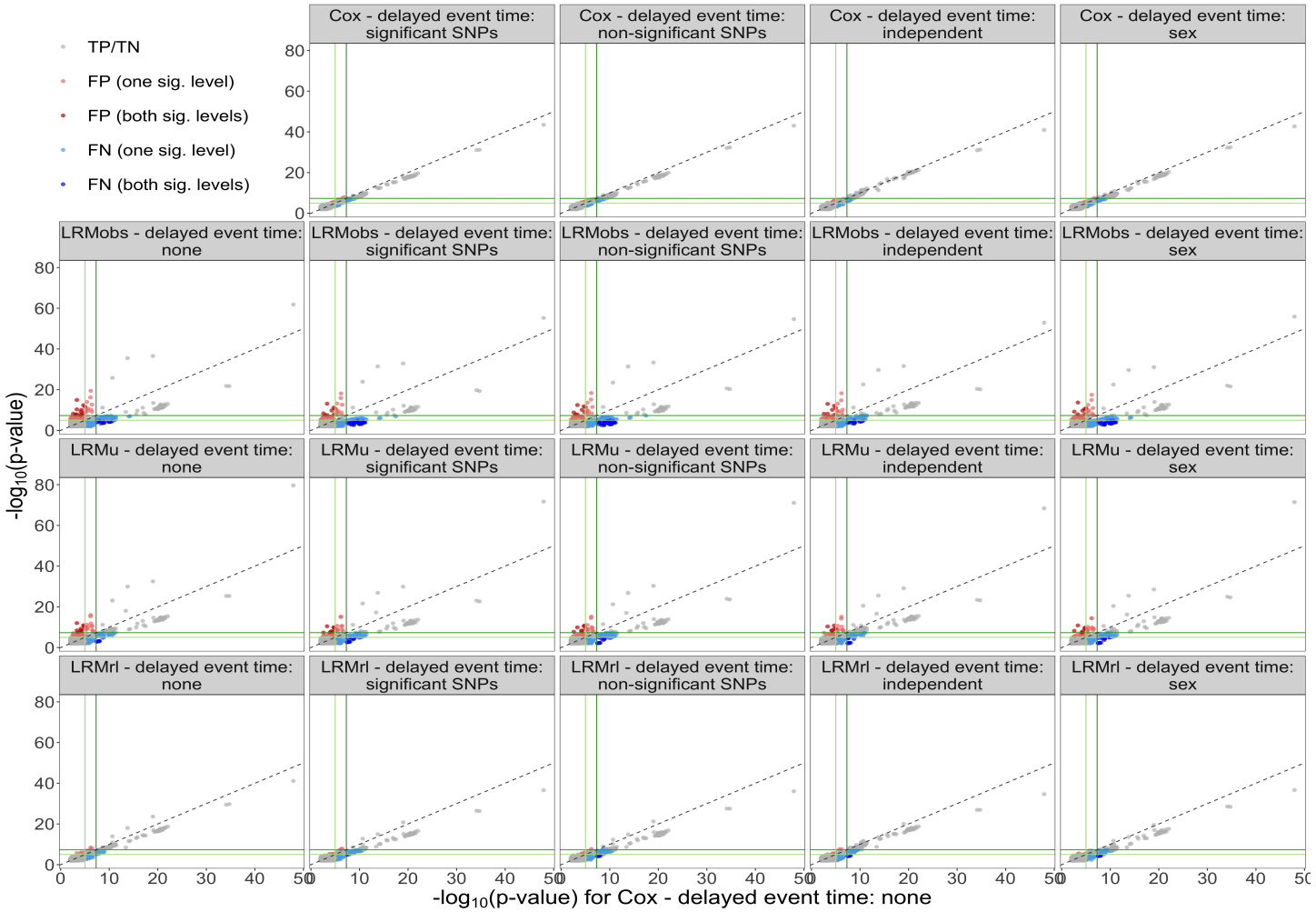

Fig. S32: False positive and false negative SNPs for each model and delayed event time combination, using Model 1 (*Cox*) with no delayed event time as the gold standard, for coronary atherosclerosis (phecode 411.4). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.

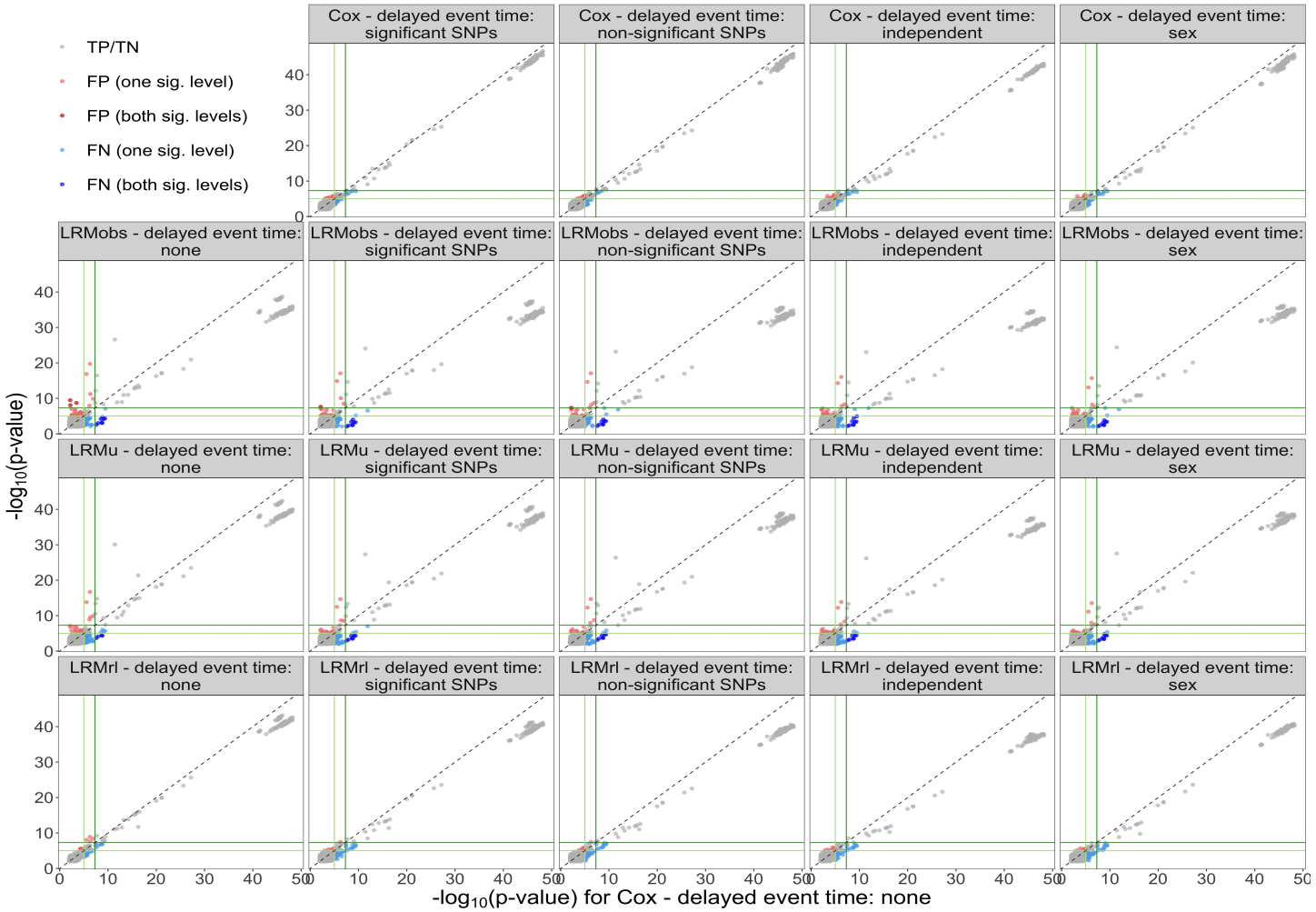

Fig. S33: False positive and false negative SNPs for each model and delayed event time combination, using Model 1 (Cox) with no delayed event time as the gold standard, for atrial fibrillation (phecode 427.21). Dark green lines correspond to $P \leq 5 \times 10^{-8}$ and light green lines correspond to $P \leq 1 \times 10^{-5}$.